C. Skin Depth

Suppose we have a plane wave. It comes from the $-z$ direction and reaches a large conductor surface at $z = 0$. Outside of a conductor: $\overrightarrow{E} = E_0 e^{-i\omega t}\vec{e}_x$ at $z = 0$.

Assume the wave inside the conductor has the form

$$\overrightarrow{E} = \overrightarrow{E}_0 e^{i(kz-\omega t)}$$

where k is an unknown constant. Recall

$$\nabla \to ik, \quad \frac{\partial}{\partial t} \to -i\omega$$

for the waves of the above type, we find from the diffusion equation

$$(ik)^2 \overrightarrow{E} = -i\omega\mu\sigma \overrightarrow{E} \quad \to \quad k^2 = i\omega\mu\sigma = \omega\mu\sigma e^{\frac{\pi}{2}i}$$

$$k = \pm e^{\frac{\pi}{4}i}\sqrt{\omega\mu\sigma} = \pm(1+i)\sqrt{\frac{\omega\mu\sigma}{2}}$$

Choose "+" sign to allow the electric field to damp (to "propagate") in the $+z$ direction. Separate the real and imaginary parts of k:

$$k = k_+ + ik_-, \qquad k_+ = k_- = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\overrightarrow{E} = \overrightarrow{E}_0 e^{-k_- z} e^{i(k_+ z - \omega t)} \tag{1}$$