$$
\begin{aligned}
&\psi && = \textcircled{c} \\
&\int_V \rho \, \frac{f(r)}{r} \, dV && = \textcircled{c} \\
&\int_0^{2\pi} \int_0^{\pi} \int_{b'}^{b} \rho \, \frac{f(r)}{r} \, a^2 \, da \sin\theta \, d\theta \, d\phi && = \textcircled{c} \\
&\int_0^{2\pi} d\phi \int_0^{\pi} \int_{b'}^{b} g(a) \, \frac{f(r)}{r} \, a^2 \, da \sin\theta \, d\theta && = \textcircled{c} \\
&p \int_0^{\pi} \left(\int_{b'}^{b} g(a) \, a \, \frac{f(r)}{r} a \, da \right) \sin\theta \, d\theta && = \frac{\textcircled{c}}{2\pi} \, p \\
&\int_0^{\pi} \left(\int_{b'}^{b} g(a) \, a \, \frac{f(r)}{r} a \, p \sin\theta \, da \right) d\theta && = \textcircled{d} \, p \\
&\int_{b'}^{b'} \left(\int_0^{\pi} g(a) \, a \, \frac{f(r)}{r} a \, p \sin\theta \, d\theta \right) da && = \textcircled{d} \, p \\
&\int_{b'}^{b} g(a) \, a \left(\int_0^{\pi} f(r) \frac{a \, p \sin\theta}{r} d\theta \right) da && = \textcircled{d} \, p \\
&\frac{\partial}{\partial q} \int_{q'}^{q} \int_{b'}^{b} g(a) \, a \left(\int_0^{\pi} f(r) \frac{a \, p \sin\theta}{r} d\theta \right) da \, dp && = \frac{\partial}{\partial q} \int_{q'}^{q} \textcircled{d} \, p \, dp \\
&\frac{\partial}{\partial q} \int_{q'}^{q} \int_{b'}^{b} g(a) \, a \left(\int_0^{\pi} f(r) \frac{a \, p \sin\theta}{r} d\theta \right) da \, dp && = \textcircled{d} \, q
\end{aligned}
\tag{1.1}
$$

We know:

$$r = \sqrt{a^2 + p^2 - 2 \, a \, p \cos\theta}$$

Also let:

$$s = a$$

$$t = p$$

Consider the expression:

$$\int_{q'}^{q} \int_{b'}^{b} g(s) \, s \left(\int_{s-t}^{s+t} f(r) \, dr \right) ds \, dt$$

We next have to change the variables from (r,s,t) to (θ,a,p) in the above expression:

$$\mathrm{J}_{(r,s,t)\to(\theta,a,p)} = \begin{bmatrix} \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{\theta}} & \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{a}} & \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{p}} \\ \frac{\partial \boldsymbol{s}}{\partial \boldsymbol{\theta}} & \frac{\partial \boldsymbol{s}}{\partial \boldsymbol{a}} & \frac{\partial \boldsymbol{s}}{\partial \boldsymbol{p}} \\ \frac{\partial \boldsymbol{t}}{\partial \boldsymbol{\theta}} & \frac{\partial \boldsymbol{t}}{\partial \boldsymbol{a}} & \frac{\partial \boldsymbol{t}}{\partial \boldsymbol{p}} \end{bmatrix} = \frac{\partial r}{\partial \theta} = \frac{a\ p \sin\ \theta}{\sqrt{a^2+p^2-2\ a\ p\cos\ \theta}}$$

$$\int_{q'}^{q}\int_{b'}^{b} g(s)\ s\left(\int_{s-t}^{s+t} f(r)\,dr\right)ds\,dt$$

$$= \int_{q'}^{q}\int_{b'}^{b} g(a)\ a\left(\int_{0}^{\pi} f(r)\ \ \mathrm{J}\ d\theta\right)da\,dp$$

$$= \int_{q'}^{q}\int_{b'}^{b} g(a)\ a\left(\int_{0}^{\pi} f(r)\frac{a\,p\sin\theta}{r}d\theta\right)da\,dp \tag{1.2}$$

By (1.1):

$$\frac{\partial}{\partial q}\int_{q'}^{q}\int_{b'}^{b} g(a)\ a\left(\int_{0}^{\pi} f(r)\frac{a\,p\sin\theta}{r}d\theta\right)da\,dp = \textcircled{d}\ q$$

By (1.2):

$$\frac{\partial}{\partial q}\int_{q'}^{q}\int_{b'}^{b} g(s)\ s\left(\int_{s-t}^{s+t} f(r)\,dr\right)ds\,dt \qquad = \textcircled{d}\ q$$

$$\frac{\partial}{\partial q}\int_{q'}^{q}\left[\int_{b'}^{b} g(s)\ s\left(\int_{s-t}^{s+t} f(r)\,dr\right)ds\right]dt \qquad = \textcircled{d}\ q$$

$$\int_{b'}^{b} g(s)\ s\left(\int_{s-q}^{s+q} f(r)\,dr\right)ds \qquad = \textcircled{d}\ q$$

$$\frac{\partial}{\partial b}\int_{b'}^{b} g(s)\ s\left(\int_{s-q}^{s+q} f(r)\,dr\right)ds \qquad = 0$$

$$\frac{\partial}{\partial b}\int_{b'}^{b}\left[g(s)\ s\left(\int_{s-q}^{s+q} f(r)\,dr\right)\right]ds \qquad = 0$$

$$g(b)\, b\, \left(\int_{b-q}^{b+q} f(r)\, dr\right) = 0$$

$$\int_{b-q}^{b+q} f(r)\, dr = 0 \tag{1.3}$$

$$\int_{b-q}^{b+q} f(r)\, dr = F(b+q) - F(b-q)$$

$$= \Big[\, F(b+q) - F(b) \,\Big] - \Big[\, F(b-q) - F(b) \Big]$$

$$= \int_{b}^{b+q} f(b+r)\, d(b+r) - \int_{b}^{b-q} f(b-r)\, d(b-r)$$

$$= \int_{0}^{q} f(b+r)\, \frac{d(b+r)}{dr}\, dr - \int_{0}^{q} f(b-r)\, \frac{d(b-r)}{dr}\, dr$$

$$= \int_{0}^{q} f(b+r)\, dr + \int_{0}^{q} f(b-r)\, dr$$

$$= \int_{0}^{q} \Big[f(b+r) + f(b-r) \Big] dr \tag{1.4}$$

By (1.3):

$$\int_{b-q}^{b+q} f(r)\, dr = 0$$

By (1.4):

$$\int_{0}^{q} \Big[f(b+r) + f(b-r) \Big] dr = 0$$

$$\frac{\partial}{\partial q} \int_{0}^{q} \Big[f(b+r) + f(b-r) \Big] dr = 0$$

$$f(b+q) + f(b-q) = 0$$

$$\frac{\partial [f(b+q)]}{\partial q} + \frac{\partial [f(b-q)]}{\partial q} = 0$$

$$f'(b+q)\,\frac{\partial(b+q)}{\partial q} + f'(b-q)\,\frac{\partial(b-q)}{\partial q} \qquad = 0$$

$$f'(b+q) - f'(b-q) \qquad = 0$$

$$f'(b+q) \qquad = f'(b-q) \tag{1.5}$$

a and p are independent variables

b and q are independent variables

$$\forall\, r \in \mathbb{R}, \exists(b, q \in \mathbb{R}) \ni \left\{\; b + q = 1 \wedge b - q = r \;\right\} \tag{1.6}$$

Statements (1.5) and (1.6) are true

$$f'(1) \qquad = f'(r)$$

$$f'(r) \qquad = ⓔ$$

$$f(r) \qquad = ⓔ r + ⓕ$$

$$u(r) = \frac{f(r)}{r} = ⓔ + \frac{ⓕ}{r} \tag{1.7}$$

By (1.7):

$$F(r)\,\hat{r} = -\boldsymbol{\nabla} u(r)$$

$$= -\boldsymbol{\nabla}\left[ⓔ + \frac{ⓕ}{r}\right]$$

$$= -ⓕ\,\boldsymbol{\nabla}\left(\frac{1}{r}\right) = ⓕ\,\frac{\hat{r}}{r^2}$$