1 General system

$$[c]_{\mathrm{SI}} = \frac{\mathrm{m}}{\mathrm{s}}$$

$$\therefore m = [c]_{\mathrm{SI}} s = \frac{c}{\{c\}_{\mathrm{SI}}} \cdot \mathrm{s} = \frac{1}{3 \cdot 10^8} \cdot c\mathrm{s}$$

Note that with all unit systems, there are exactly two restrictions with dimensionless constants:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \approx \frac{1}{137.04}$$

$$\gamma = \frac{G m_e^2}{\hbar c} \approx 1.751 \cdot 10^{-45} = 2.40000 \cdot 10^{-43} \cdot \alpha$$

You see that the electromagnetic and gravitational forces can be states in terms of $\hbar c$.

2 System: SI

$$
\begin{aligned}
c &\overset{\text{def}}{=} \mathbf{2.99792458 \cdot 10^{8}}\ \mathrm{m\,s^{-1}} & m_e &= \mathbf{9.1093897 \cdot 10^{-31}}\ \mathrm{kg} & N_A &= \mathbf{6.022136 \cdot 10^{23}}\ \mathrm{mol^{-1}} = 84446888^3 \\
G &= \mathbf{6.67259 \cdot 10^{-11}}\ \mathrm{m^3\,kg^{-1}\,s^{-2}} & m_n &= \mathbf{1.6749286 \cdot 10^{-27}}\ \mathrm{kg} & V_n &= \mathbf{22.414}\ \mathrm{l\,mol^{-1}} \\
\varepsilon_0 &= \mathbf{8.854188 \cdot 10^{-12}}\ \mathrm{A\,s\,V^{-1}\,m^{-1}} & m_p &= \mathbf{1.6726231 \cdot 10^{-27}}\ \mathrm{kg} & p_n &= \mathbf{101325}\ \mathrm{Pa} \\
\mu_0 &= \mathbf{1.256637 \cdot 10^{-6}}\ \mathrm{V\,s\,A^{-1}\,m^{-1}} & 1\mathrm{u} &= \mathbf{1.660540 \cdot 10^{-27}}\ \mathrm{kg} & g_n &= \mathbf{9.80665}\ \mathrm{m\,s^{-2}} \\
\hbar &= \mathbf{1.054573 \cdot 10^{-34}}\ \mathrm{J\,s} & & & T_n &= \mathbf{273.15}\ \mathrm{K} \\
e &= \mathbf{1.602177 \cdot 10^{-19}}\ \mathrm{As} & & & & \\
k_B &= \mathbf{1.380658 \cdot 10^{-23}}\ \mathrm{J\,K^{-1}} & & & &
\end{aligned}
$$

3 System: eV, Å/cm, GHz, T

eV	thermo (T)	magnetic (B)	light (ν)	light ($\overline{\nu}$)
1 meV	$k_B \cdot$ **11.60** K	$\mu_B \cdot$ **17.28** T	$h \cdot$ **241.8** GHz	$hc \cdot$ **8.066** cm^{-1}
0.08617 meV	$k_B \cdot$ **1** K	$\mu_B \cdot$ **1.489** T	$h \cdot$ **20.84** GHz	$hc \cdot$ **0.6960** cm^{-1}
0.05788 meV	$k_B \cdot$ **0.6717** K	$\mu_B \cdot$ **1** T	$h \cdot$ **14.00** GHz	$hc \cdot$ **0.4669** cm^{-1}
4.136 meV	$k_B \cdot$ **47.99** K	$\mu_B \cdot$ **71.45** T	$h \cdot$ **1000** GHz	$hc \cdot$ **33.36** cm^{-1}
0.1240 meV	$k_B \cdot$ **1.439** K	$\mu_B \cdot$ **2.142** T	$h \cdot$ **29.98** GHz	$hc \cdot$ **1** cm^{-1}
13.6 eV	$k_B \cdot$ **157820** K	$\mu_B \cdot$ **234953** T	$h \cdot$ **3289** THz	$hc \cdot$ **109691** cm^{-1}
23.54 meV	$k_B \cdot$ **273.15** K	$\mu_B \cdot$ **406.6** T	$h \cdot$ **5692** GHz	$hc \cdot$ **189.8** cm^{-1}
25.43 meV	$k_B \cdot$ **295.15** K	$\mu_B \cdot$ **439.4** T	$h \cdot$ **6150** GHz	$hc \cdot$ **205.1** cm^{-1}

Table 1: Energy conversion

$$1\ \text{meV} = \frac{hc}{\mathbf{0.1240}\text{cm}}$$
$$1\ \text{GHz} = \frac{c}{\mathbf{29.98}\text{cm}}$$

$$1\ \text{kg} = \mathbf{6.242} \cdot \mathbf{10^{16}}\ \text{eV}\,\text{GHz}^{-2}\text{Å}^{-2}$$
$$1\ \text{m} = \mathbf{10^{10}}\ \text{Å}$$
$$1\ \text{s} = \mathbf{10^{9}}\ \text{GHz}^{-1}$$

$$1\ \text{eV} = \mathbf{1.602177} \cdot \mathbf{10^{-19}}\ \text{J}$$
$$1\ \text{A} = \mathbf{0.624}\ \text{eV}\text{T}^{-1}\text{Å}^{-2}$$
$$1\ \text{V} = \mathbf{10^{11}}\ \text{TÅ}^2\text{GHz}$$

$$c = \mathbf{29.98}\ \text{cm}\,\text{GHz}$$
$$\hbar = \mathbf{6.582} \cdot \mathbf{10^{-7}}\ \text{eV}\,\text{GHz}^{-1}$$
$$\varepsilon_0 = \mathbf{5.526} \cdot \mathbf{10^{-25}}\ \text{eV}\,\text{GHz}^{-2}\text{T}^{-2}\text{Å}^{-5}$$
$$\mu_0 = \mathbf{201335}\ \text{T}^2\text{Å}^3\text{eV}^{-1}$$
$$e = \mathbf{10^{-11}}\ \text{eV}\,\text{GHz}^{-1}\ \text{Å}^{-2}\text{T}^{-1}$$
$$k_B = \mathbf{0.08617}\ \text{meVK}^{-1}$$

$$m_e = \mathbf{5.6856} \cdot \mathbf{10^{-14}}\ \text{eV}\,\text{GHz}^{-2}\text{Å}^{-2}$$
$$m_n = \mathbf{1.0454} \cdot \mathbf{10^{-10}}\ \text{eV}\,\text{GHz}^{-2}\text{Å}^{-2}$$
$$m_p = \mathbf{1.0440} \cdot \mathbf{10^{-10}}\ \text{eV}\,\text{GHz}^{-2}\text{Å}^{-2}$$
$$1\text{u} = \mathbf{1.0364} \cdot \mathbf{10^{-10}}\ \text{eV}\,\text{GHz}^{-2}\text{Å}^{-2}$$

$$\frac{\hbar^2}{2m_e} = \mathbf{3.810}\ \text{eVÅ}^2$$
$$\frac{e^2}{4\pi\varepsilon_0} = \mathbf{14.40}\ \text{eVÅ}$$
$$\frac{e}{m_e} = \mathbf{175.9}\ \text{GHz}\,\text{T}^{-1}$$

$$\Phi_0 = \frac{h}{2e} = \mathbf{206783}\ \text{TÅ}^2$$
$$\frac{2\pi e}{\hbar} = \mathbf{9.546} \cdot \mathbf{10^{-5}}\ \text{T}^{-1}\text{Å}^{-2}$$
$$\mu_B = \frac{e\hbar}{2m_e} = \mathbf{0.05788}\ \text{meV}\,\text{T}^{-1}$$

$$\frac{\hbar^2}{2m_n} = \mathbf{2.072}\ \text{meVÅ}^2$$
$$\mu_N = \frac{e\hbar}{2m_n} = \mathbf{3.1481} \cdot \mathbf{10^{-5}}\ \text{meV}\,\text{T}^{-1}$$

$$E_R = \frac{1}{2}\mu(c\alpha)^2 = \mathbf{13.6}\ \text{eV}$$
$$a_0 = \frac{\hbar}{\mu(c\alpha)} = \mathbf{0.529}\ \text{Å}$$

4 Electronic system

The new pseudo-units are:

$$\hbar = \varepsilon_0 = e = m_e = k_B = \mathbf{1}$$

$$1\ \text{Å} = \mathbf{23.75} \cdot \frac{\hbar^2\varepsilon_0}{m_e e^2} \qquad 1\ \text{K} = \mathbf{2.005 \cdot 10^{-8}} \cdot \frac{e^4 m_e}{\varepsilon_0^2\hbar^2 k_B}$$

$$1\ \text{meV} = \mathbf{0.2327} \cdot \frac{e^4 m_e}{\varepsilon_0^2\hbar^2} \qquad 1\ \text{T} = \mathbf{2.694 \cdot 10^{-8}} \cdot \frac{e^3 m_e^2}{\varepsilon_0^2\hbar^3}$$

$$1\ \text{s} = \mathbf{6.528 \cdot 10^{18}} \cdot \frac{\hbar^3\varepsilon_0^2}{m_e e^4} \qquad 1\ \text{A} = \mathbf{0.9560} \cdot \frac{e^5 m_e}{\varepsilon_0^2\hbar^3}$$

$$1\ \text{kg} = \mathbf{1.098 \cdot 10^{30}} \cdot m_e \qquad 1\ \text{V} = \mathbf{2.327 \cdot 10^{-4}} \cdot \frac{e^3 m_e}{\varepsilon_0^2\hbar^2}$$

$$c = \frac{1}{4\pi\alpha} = \mathbf{10.90} \qquad m_n = \mathbf{1839}$$

$$G = \mathbf{2.331 \cdot 10^{23}} \qquad m_p = \mathbf{1836}$$

$$\mu_0 = (4\pi\alpha)^2 = \mathbf{8.409 \cdot 10^{-3}} \qquad 1\text{u} = \mathbf{1822}$$

$$\Phi_0 = \pi \qquad E_R = \frac{1}{2}\mu(c\alpha)^2 \approx \mathbf{3.166 \cdot 10^{-3}}$$

$$22^\circ\text{C} = \mathbf{4.672 \cdot 10^{-4}} \qquad a_0 = \frac{\hbar}{\mu(c\alpha)} \approx \mathbf{4}\pi$$

The Schrödinger equation is

$$\frac{1}{2}\nabla^2\Psi + \frac{1}{4\pi r^2}\Psi = -\mathrm{i}\frac{\partial\Psi}{\partial t}$$

Since we leave out the constant that are numerically 1, we need to reintroduce them on backconversion. In an equation k means $\{k\} = k/[k]$. Therefore the backconversion is done via

$$r \to r\left(\frac{\hbar^2\varepsilon_0}{m_e e^2}\right)^{-1}$$

$$k \to k\left(\frac{m_e e^2}{\hbar^2\varepsilon_0}\right)^{-1}$$

$$E \to E\left(\frac{e^4 m_e}{\varepsilon_0^2\hbar^2}\right)^{-1}$$

and so on.

5 Units for the Schrödinger equation

$$\hbar = 4\pi\varepsilon_0 = e = 2m_e = k_B = \mathbf{1}$$
$$\therefore \frac{\hbar^2}{2m_e} = \frac{e^2}{4\pi\varepsilon_0} = \mu_B = \mathbf{1}$$

$$\begin{aligned} 1\,\text{Å} &= \mathbf{3.779} & 1\,\text{K} &= \mathbf{1.583 \cdot 10^{-6}} \\ 1\,\text{meV} &= \mathbf{18.37} & 1\,\text{T} &= \mathbf{1.0636 \cdot 10^{-6}} \\ 1\,\text{s} &= \mathbf{8.268 \cdot 10^{16}} & 1\,\text{A} &= \mathbf{75.49} \\ 1\,\text{GHz} &= \mathbf{1.209 \cdot 10^{-8}} & 1\,\text{V} &= \mathbf{0.01837} \\ 1\,\text{kg} &= \mathbf{5.489 \cdot 10^{29}} \end{aligned}$$

$$\begin{aligned} c &= \frac{1}{\alpha} = \mathbf{137.04} & m_n &= \mathbf{919.3} \\ G &= \mathbf{4.638 \cdot 10^{21}} & m_p &= \mathbf{918.1} \\ \mu_0 &= 4\pi\alpha^2 = \mathbf{6.692 \cdot 10^{-4}} & 1\text{u} &= \mathbf{911.4} \end{aligned}$$

$$\begin{aligned} \Phi_0 &= \pi & E_R &= \frac{1}{2}\mu(c\alpha)^2 \approx \mathbf{0.5} \\ 22^\circ\text{C} &= \mathbf{5.92 \cdot 10^{-6}} & a_0 &= \frac{\hbar}{\mu(c\alpha)} \approx \mathbf{0.5} \end{aligned}$$

The Schrödinger equation is

$$\nabla^2\Psi + \frac{1}{r^2}\Psi = -\mathrm{i}\frac{\partial\Psi}{\partial t}$$

For back-substitution see previous section with $m_e \to 2m_e$ and $\varepsilon_0 \to 4\pi\varepsilon_0$.