Mixed Turkish Tubitak olympiads problems and solutions

Rafiq Abbasov

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Problems

1 Problem 1

There is a plane with slope angle θ.Length of the first part's plane is l_1.Friction ratio of l_1 is μ_1.The length of the second part's plane is l_2 and its friction ratio is μ_2.We know that $\mu_1 > \mu_2$.Object replaced on the top of the plane.If an object falls down and stops bottom of the plane what'ratio between l_1 and l_2?

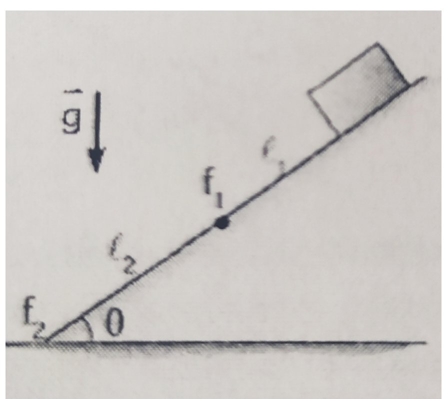

Figure 1: Problem 1

2 Problem 2

Electron with mass m collides with nuclear with mass M.Because of the collision combined nuclear gets energy with magnitude E.What was the velocity of electron?(Ignore relativistic effects)

3 Problem 3

The rocket sent to space from planet with velocity v_0.What is the maximum altitude rocket can rise?(Radius of the planet is R and ignore air resistance)

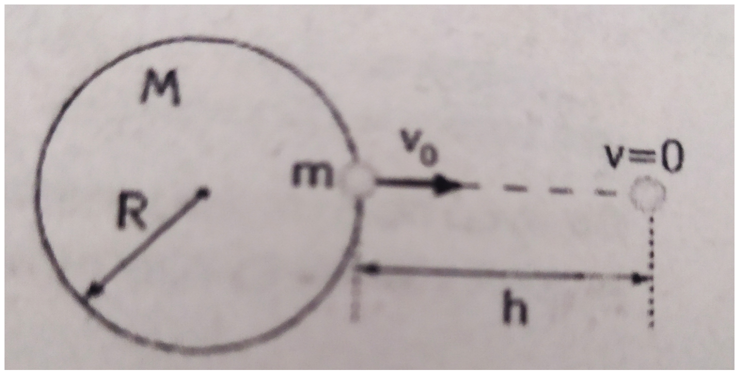

Figure 2: Problem 3;

4 Problem 4

There is a huge container with 2 bases.Area of above base is A_1,below base's is A_2.These bases closed by active pistons.Masses of pistons are very small so we can ignore them.Also pistons connected with each other with the help of string in length l.Find the tension force appears in string.(Ignore friction.There is also atmosphere pressure effects to bases.g is gravity acceleration)

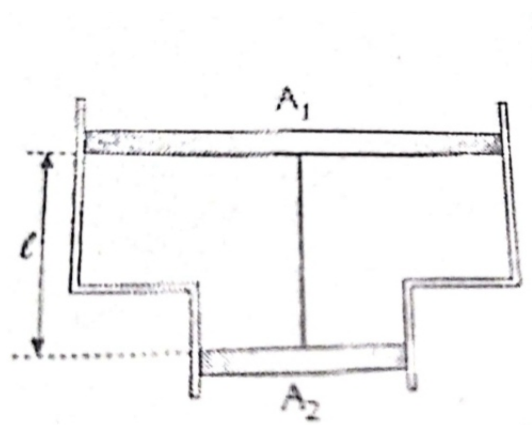

Figure 3: Problem 4

5 Problem 5

1 mol monoatomic ideal gas does some prosses.$A-B$ is adiabatic prosses.Find the efficiency of cycle.($\gamma = 5/3$)

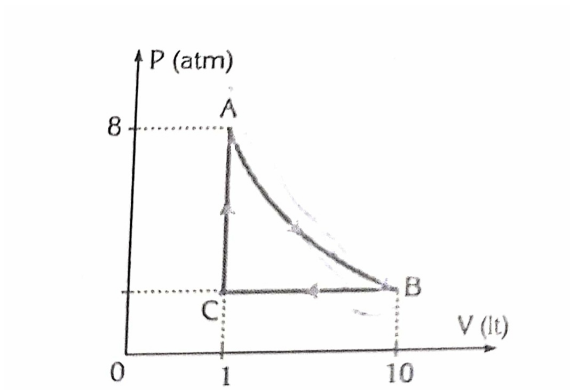

Figure 4: Problem 5

Solutions

6 Problem 1

The road which object went is

$$l_1 = \frac{v^2}{2a_1}$$
$$l_2 = \frac{v^2}{2a_2}$$

Thus:

$$l_1 a_1 = l_2 a_2$$
$$a_1 = g(sin\theta - \mu_1 cos\theta)$$
$$a_2 = g(\mu_2 cos\theta - sin\theta)$$

Dividing these yields gives us:

$$\frac{l_1}{l_2} = \frac{\mu_2 - tan\theta}{tan\theta - \mu_1}$$

7 Problem 2

Initially momentum of electron is $P = mv$, momentum of combined nuclear and electron is $P_{combined} = (m+M)v_{middle}$ From balance of energy we can write:

$$\frac{1}{2}mv^2 = \frac{1}{2}(m+M)v_{middle}^2 + E$$

These 2 equations give us:

$$v = \sqrt{\frac{2E(m+M)}{mM}}$$

8 Problem 3

We know that total initially energy of system is:

$$E_0 = \frac{1}{2}mv_0^2 - \frac{GMm}{R}$$

$$E_{final} = \frac{-GMm}{R+h}$$

For energy balance we can write:

$$\frac{1}{2}mv_0^2 - \frac{GMm}{R} = \frac{-GMm}{R+h}$$

We know that $GM = gR^2$ So:

$$\frac{1}{2}v_0^2 - \frac{GR^2}{R} = \frac{-gR^2}{R+h}$$

This gives us:

$$h = \frac{v_0^2 R}{2gR - v_0^2}$$

9 Problem 4

Let's say pressure which effects above piston P,so below piston's pressure must be $P = P_0 + \rho gl$ We know that pistons are balanced.And we can write:

$$P_0 A_1 + T = PA_1$$

$$(P + \rho gl)A_2 = P_0 A_2 + T$$

These give us:

$$T = \frac{\rho gl A_1 A_2}{A_1 - A_2}$$

10 Problem 5

We know that $A - B$ is adiabatic prosses.First of all we can find magnitude of P_C otherwise P_B.We can write :

$$P_{AV}^{\gamma} = P_{BV}^{\gamma}$$

solving this yields for P_B we get

$$P_B = P_A \frac{V_A^\gamma}{V_B^\gamma}$$

Now we gotta find efficiency of cycle.$\eta = 1 - \frac{Q_{giv}}{Q_{got}}$.$Q_{got}$ is $A - B$ and $C - B$ prosses so: For the 1^{st} rule of thermodynamics $Q = W + U$

$$Q_{BC} = P\delta V + \frac{3}{2} nR\delta T$$

$$Q_{BC} = \frac{5}{2} nR\delta T$$

So we can write:

$$Q_{BC} = \frac{5}{2} \delta PV$$

But law for $A - B$ is a little different.We know that it's adiabatic prosses.So we can write: $\delta * U = 0$ but work is $W = \int_1^{10} PdV$.It gives us $P\delta V$ For prosses $A - C$ we can write only $Q = \frac{3}{2} nR\delta T$ also we can write $Q = \frac{3}{2} \delta PV$ At the end:

$$\eta = 1 - \frac{1.5(8 - 0.17)}{(8 - 0.17)9 + 2.5 * 0.17 * 9}$$

The answer is $\eta = 0.84$