

Note to self: The torus can be thought of as a product of the sets  $\{y^2 + x^2 = 1\}$  and  $\{w^2 + r^2 = 1\}$ . We imagine a torus as a circle rotated around in a circle. The first equation defines our location on the first circle, and the second the second circle.

(3.c) We recall the image of  $(S^1 \times S^1)$  is covered twice under  $F$ . The ~~length~~ length of the tangent vector, we can think of as speed. Our intuition is that  $F$  doubles the lengths of all tangent vectors, to cover ~~area~~ twice the area in the same time.

We can compare  $\sqrt{dp^2 + d\varphi^2 + dr^2 + ds^2}$  and  $\sqrt{dw^2 + dx^2 + dy^2 + dz^2}$ .

$$\text{Substitute } \begin{cases} dp = 2p \\ d\varphi = 2\varphi \\ dr = 2r \\ ds = 2s \end{cases} \text{ and choose a coordinate } \begin{cases} p = \frac{1}{\sqrt{2}} \\ \varphi = \frac{1}{\sqrt{2}} \\ r = \frac{1}{\sqrt{2}} \\ s = \frac{1}{\sqrt{2}} \end{cases} \text{ which is convenient.}$$

$$\text{we have } \sqrt{\left(\frac{2}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2} = \sqrt{8} = (\sqrt{dp^2 + d\varphi^2 + dr^2 + ds^2})^{1/2}$$

$$\text{and } \sqrt{2\left(\frac{2}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{2}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}\right)^2} \times 4 \Rightarrow \sqrt{32} = (\sqrt{dw^2 + dx^2 + dy^2 + dz^2})^{1/2}$$

To find the factor of length expansion by  $F$ , we set  $\frac{(\sqrt{dp^2 + d\varphi^2 + dr^2 + ds^2})^{1/2}}{(\sqrt{dw^2 + dx^2 + dy^2 + dz^2})^{1/2}}$

$$\Rightarrow \sqrt{32/8} \Rightarrow \sqrt{4} \Rightarrow 2$$