Carried by impulse: The physics of water jetpacks

Matthew Vonk and Peter Bohacek

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Carried by impulse: The physics of water jetpacks

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A jetpack can suspend a pilot in midair, in a way reminiscent of a magic carpet. But the magic is all in mechanical forces that can be readily estimated.

Matt Vonk is a physics professor at the University of Wisconsin–River Falls. **Peter Bohacek** is a physics teacher at Henry Sibley High School in Mendota Heights, Minnesota.

One morning I (Vonk) happened across a local news story about a water-driven jetpack and was immediately intrigued. The motion provided by the craft is intuitive and organic in a way that closely resembles the unencumbered flight that many of us dreamt about as kids. If you haven't already seen a jetpack in action, you can check out how they work at http://www.youtube.com/watch?v=Cd6C1vIyQ3w.

And yet the technology isn't new. The jetpacks are powered via a fire hose that is easily attached to the output jet of a personal watercraft. When the water in the hose reaches the flying platform, it is split into two large thrusters, one below each foot, and two smaller handheld stabilizers.

I had a chance to see the evocative craft in person one gorgeous late-summer afternoon when local jetpack phenom Caleb Gavic was flying above the Mississippi River near downtown Minneapolis. Watching Gavic, I was simultaneously struck by a number of unrelated reactions. The five-year-old in me was enthralled by the coolness of his movements, some of which are caught in the figure. The fearless adolescent in me wanted to jump on and give it a try. But strongest of all was the geeky physics-teacher compulsion to puzzle out the physical principles that permitted his gravity-defying flight.

A first pass at the mechanics

For simplicity, I started with a one-dimensional approximation. When Gavic is stationary, hovering above the river's surface, the force exerted by the expelled water must be enough to support the weight of the pilot and the platform, which in his case is 910 N. Since $F = dp/dt$, the supported weight must be equal to the change in momentum per unit time of the water at the platform. In the steady state, the flow rate of the water in the hose is constant, so dp/dt is the product of the mass of water that approaches the platform per unit time, dm/dt, and the change in the velocity of the water:

$$F = dp/dt = dm/dt\,(v_{\mathrm{f}} - v_{\mathrm{i}})\,, \tag{1}$$

where v_{i} and v_{f} are the initial (in the hose) and final (expelled) water velocities.

In terms of measurable quantities, the mass change dm/dt can be expressed as $dm/dt = \rho\, v_{\mathrm{f}}\,(2A_{\mathrm{T}} + 2A_{\mathrm{S}})$, where ρ is the mass density of water, A_{T} is the cross-sectional area of each of the thrusting nozzles, and A_{S} is the area of each of the two handheld stabilizing nozzles. Notice that the final speed of the water will be greater than the initial speed, because the nozzles have a smaller total cross-sectional area than the fire hose. Neglecting the water that spills out around the platform's ball bearings, the volume of water that approaches the platform in a given time must be the same as the volume of water that leaves the nozzles, so

$$v_{\mathrm{i}} = -v_{\mathrm{f}}(2A_{\mathrm{T}} + 2A_{\mathrm{S}})/A_{\mathrm{H}}. \tag{2}$$

Here A_{H} is the cross-sectional area of the fire hose, and the minus sign arises because the final and initial velocities are oppositely directed. Combining the results obtained so far yields for the force

$$F = \rho\, v_{\mathrm{f}}^{\,2}\,(2A_{\mathrm{T}} + 2A_{\mathrm{S}})\,[1 + (2A_{\mathrm{T}} + 2A_{\mathrm{S}})/A_{\mathrm{H}})]\,. \tag{3}$$

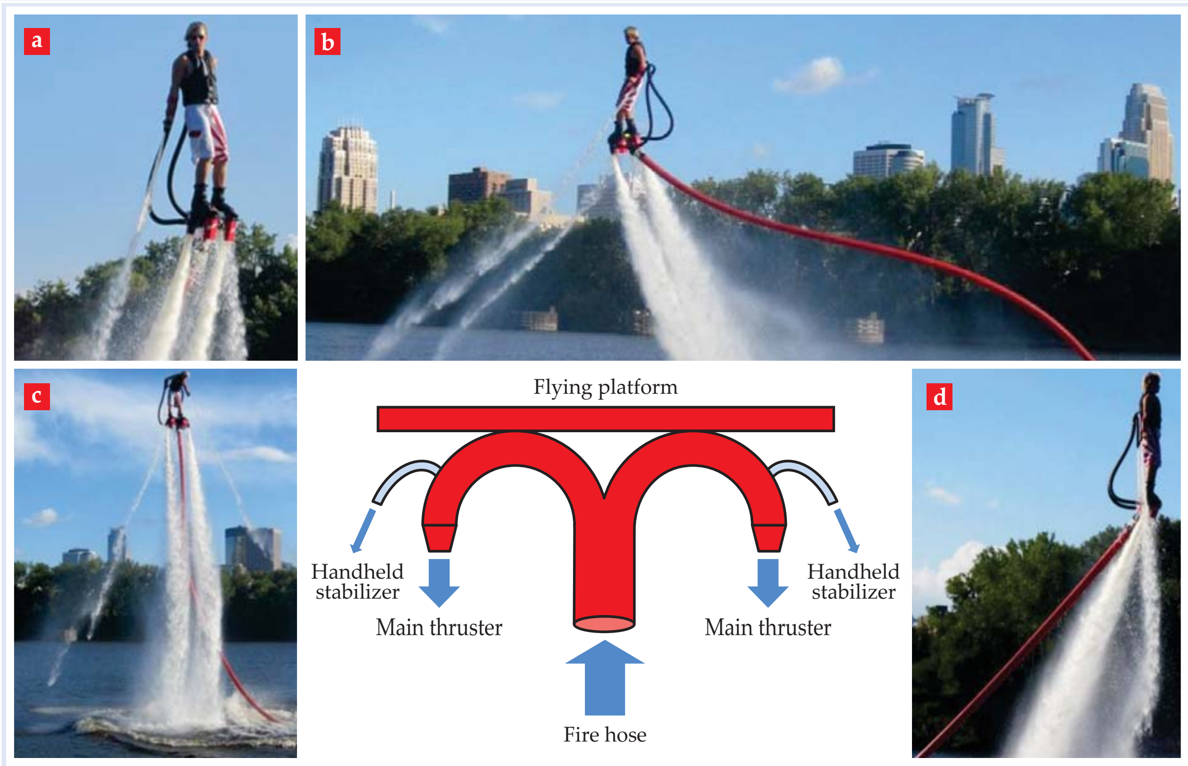
Plugging in the known values $F = 910$ N, $\rho = 1000$ kg/m^3, $A_{\mathrm{T}} = 2.3 \times 10^{-3}$ m^2, $A_{\mathrm{S}} = 2.9 \times 10^{-4}$ m^2, and $A_{\mathrm{H}} = 8.1 \times 10^{-3}$ m^2 allows one to solve for the minimum value of v_{f}, which turns out to be a bit over 10 m/s, about 23 mph.

The tension mounts

Peter Bohacek and I were curious to test my 10 m/s answer experimentally, so we took a high-speed video of Gavic hovering above the water. That video, along with a standard-speed video of the board in motion, is available in the Science Education Resource Center's "Pedagogy in Action" video library at http://serc.carleton.edu/sp/library/direct_measurement_video/video_library.html.

The knots of water ejected by the nozzles in the high-speed video take about 16 frames to travel 1 m. At 240 frames per second, that corresponds to a speed of 15 m/s. (We estimate the error in the speed determination to be ±1 m/s.) Such a speed seems reasonable for the output of a personal watercraft, if we keep in mind that the machine is not operating at full throttle. Although our measured value is in the same ballpark as the theoretical prediction, it's not as close as we would have liked. We were particularly disappointed because impulse is proportional to the velocity squared; higher water speed in the hose increases both the mass of water hitting the platform each second and the momentum transferred by each unit mass. In fact, plugging 15 m/s into equation 3 yields a total force of 1900 N—much higher than the actual weight of 910 N.

The velocity I calculated in the previous section, however, was the minimum value needed to support the weight of Gavic and the platform. In addition to gravity, the hose also pulls down on the craft with a force at least equal to the weight of the hose and its liquid contents. Indeed, the upward thrust of the jets can exceed the minimum needed for support, even taking the hose into account; in that case, the



Gliding and hovering with water jetpacks. (a) Jetpack expert Caleb Gavic pilots the flying craft (schematically drawn in the center of the figure) over the Mississippi River near downtown Minneapolis. A main thruster and a handheld stabilizer are visible in the foreground; the red hose through which water enters the craft is barely visible toward the back. Some of the water leaks at the ball bearings where entrance and exit ports are connected to the jetpack platform. **(b)** Gavic lowers himself while turning gently to his left and **(c)** hovers 10 m above the river. **(d)** This hovering pose, not as high as in panel c, is analyzed in detail in the text.

craft will accelerate upward until additional tension in the hose stops it. Although at first, quantifying the tension may seem impossible, a closer look at the hovering vehicle from the side offers a tantalizing clue.

As shown in panel d of the figure, the main thrusters and the fire hose tend to angle backward when the jetpack is hovering. To hold the pilot in one position, the horizontal component of the tension T in the fire hose must be equal and opposite to the horizontal component of force produced by the redirected water. Equating the magnitudes of those force components leads to

$$T\sin\theta_{\rm H} = \rho v_{\rm i}{}^2 A_{\rm H}\sin\theta_{\rm H} + 2\rho v_{\rm f}{}^2 A_{\rm T}\sin\theta_{\rm T}, \qquad (4)$$

where $\theta_{\rm T}$ is the angle the thrusters make with the vertical (in this case about 25°) and $\theta_{\rm H}$ is the angle the hose makes with the vertical (in this case about 45°). Solving for the tension yields a value of 1660 N.

Armed with the tension and angles involved, we now calculate the payload that the jetpack should be able to lift. In the steady-state case, the upward force provided by the water in the hose (in expression 5a to follow), the thrusters (5b), and the stabilizers (5c) should equal the downward component of the tension in the hose plus the weight W of the platform and pilot (5d):

$$\rho v_{\rm i}{}^2 A_{\rm H}\cos\theta_{\rm H} \qquad (5a)$$

$$+\, 2\rho v_{\rm f}{}^2 A_{\rm T}\cos\theta_{\rm T} \qquad (5b)$$

$$+\, 2\rho v_{\rm f}{}^2 A_{\rm S} \qquad (5c)$$

$$= T\cos\theta_{\rm H} + W. \qquad (5d)$$

Treating W as an unknown, using equation 2 to obtain $v_{\rm i}$ in terms of $v_{\rm f}$, and plugging the known values of all the remaining variables into equation 5 yields $W = 770$ N. Considering the difficulty of accurately measuring the various velocities and angles of Gavic's mercurial craft, we're pleased to be within about 15% of the actual weight of 910 N. That's especially true considering that all the terms in equation 5 except for W depend on velocity squared.

Flying high

Although we didn't get high-speed footage of Gavic hovering really high above the Mississippi, as in panel c of the figure, we were curious about what water-jet speeds would be required to support such extravagant flight. At the instant the picture was taken, Gavic's feet are about 10 m above the water. At that height, the mass of the hose and the water in it is 110 kg. The hose is hanging almost straight down, so the minimum tension in it is the weight of hose and water, 1100 N. The thrusters are also very close to vertical in panel c. Thus we can set $\theta_{\rm H}$ and $\theta_{\rm T}$ equal to zero in equation 5 and solve for the minimal $v_{\rm f}$ given the known weight of Gavic and the jetpack; the result is 15.4 m/s. But it's likely that the hose has an additional tension. To see how that additional force affects the final velocity, we set $T = 1660$ N (as before) and recalculated $v_{\rm f} = 17.4$ m/s = 39 mph. We're anxious to take another high-speed video to see how close we got, but given that it's winter in Minnesota, I think we'll have to wait a while to entice Gavic back into the water.

Now that I've worked through some jetpack physics, the five-year-old in me still gets a kick out of watching the craft fly around, and the juvenile in me still wants to risk my neck trying it. But the physicist in me is leaning back in his chair with his feet up, feeling satisfied and content. ■