Correspondingly, the roots of the proposed equation are

$$x_1 = \frac{1}{3}\left(\sqrt[3]{26 + 15\sqrt{3}} + \sqrt[3]{26 - 15\sqrt{3}} - 1\right),$$

$$x_2 = -\frac{1}{6}\left(\sqrt[3]{26 + 15\sqrt{3}} + \sqrt[3]{26 - 15\sqrt{3}} + 2\right) + \frac{i\sqrt{3}}{6}\left(\sqrt[3]{26 + 15\sqrt{3}} - \sqrt[3]{26 - 15\sqrt{3}}\right),$$

$$x_3 = -\frac{1}{6}\left(\sqrt[3]{26 + 15\sqrt{3}} + \sqrt[3]{26 - 15\sqrt{3}} + 2\right) - \frac{i\sqrt{3}}{6}\left(\sqrt[3]{26 + 15\sqrt{3}} - \sqrt[3]{26 - 15\sqrt{3}}\right).$$

The equation

$$x^3 + x^2 - 2 = 0$$

has, however, an integral root 1 and the remaining two roots,

$$-1 \pm i,$$

are imaginary.

On comparison with the expressions obtained from Cardan's formulas we discover the rather curious fact that

$$\sqrt[3]{26 + 15\sqrt{3}} + \sqrt[3]{26 - 15\sqrt{3}} = 4.$$

although both cube roots are irrational numbers. The explanation of this follows from a comparison of the imaginary roots. This comparison gives for the difference of the same cube roots

$$\sqrt[3]{26 + 15\sqrt{3}} - \sqrt[3]{26 - 15\sqrt{3}} = 2\sqrt{3},$$

whence

$$\sqrt[3]{26 + 15\sqrt{3}} = 2 + \sqrt{3}, \qquad \sqrt[3]{26 - 15\sqrt{3}} = 2 - \sqrt{3}.$$

It follows that $26 + 15\sqrt{3}$ and $26 - 15\sqrt{3}$ are cubes of numbers $2 + \sqrt{3}$ and $2 - \sqrt{3}$. Such simplification of cube roots occurs always when the cubic equation has a rational root but not otherwise.

Example 2. To solve the equation

$$x^3 + 9x - 2 = 0.$$

Here the preliminary transformation is not necessary and Cardan's formulas can be applied directly. We have

$$p = 9, \qquad q = -2, \qquad \Delta = 3024, \qquad \frac{\Delta}{108} = 28,$$

$$A = 1 + \sqrt{28}, \qquad B = 1 - \sqrt{28}.$$

Consequently, the real root is

$$\sqrt[3]{\sqrt{28} + 1} - \sqrt[3]{\sqrt{28} - 1}$$

while the imaginary roots are

$$-\frac{1}{2}\left(\sqrt[3]{\sqrt{28} + 1} - \sqrt[3]{\sqrt{28} - 1}\right) \pm \frac{i\sqrt{3}}{2}\left(\sqrt[3]{\sqrt{28} + 1} + \sqrt[3]{\sqrt{28} - 1}\right).$$