Obviously, Δ will be positive, zero, or negative. Supposing first Δ to be positive, the square root

$$\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}=\sqrt{\frac{\Delta}{108}}$$

will be real, and we shall take it positively. Then, A and B will be real, and by $\sqrt[3]{A}$ we shall mean the real cube root of A. Since p is real and

$$\sqrt[3]{A}\cdot\sqrt[3]{B}=-\frac{p}{3},$$

$\sqrt[3]{B}$ will be the real cube root of B. Hence, equation (1) has a real root

$$y_1=\sqrt[3]{A}+\sqrt[3]{B},$$

but the other two roots,

$$y_2=\omega\sqrt[3]{A}+\omega^2\sqrt[3]{B}=-\frac{\sqrt[3]{A}+\sqrt[3]{B}}{2}+i\sqrt{3}\frac{\sqrt[3]{A}-\sqrt[3]{B}}{2},$$

$$y_3=\omega^2\sqrt[3]{A}+\omega\sqrt[3]{B}=-\frac{\sqrt[3]{A}+\sqrt[3]{B}}{2}-i\sqrt{3}\frac{\sqrt[3]{A}-\sqrt[3]{B}}{2},$$

will be imaginary conjugates since A and B are not equal and consequently

$$\sqrt[3]{A}-\sqrt[3]{B}\neq 0.$$

Example 1. Let the proposed cubic equation be

$$x^3+x^2-2=0.$$

First, it must be transformed by the substitution

$$x=y-\tfrac{1}{3}.$$

The resulting equation for y (best found by synthetic division) is

$$y^3-\tfrac{1}{3}y-\tfrac{52}{27}=0,$$

so that

$$p=-\frac{1}{3},\qquad q=-\frac{52}{27},\qquad \Delta=\frac{52^2}{27}-\frac{4}{27}=100.$$

Hence,

$$\sqrt{\frac{\Delta}{108}}=\frac{5}{\sqrt{27}},\qquad A=\frac{26}{27}+\frac{5\sqrt{27}}{27},\qquad B=\frac{26}{27}-\frac{5\sqrt{27}}{27},$$

$$\sqrt[3]{A}=\tfrac{1}{3}\sqrt[3]{26+15\sqrt{3}},\quad \sqrt[3]{B}=\tfrac{1}{3}\sqrt[3]{26-15\sqrt{3}},$$

and

$$y_1=\frac{1}{3}\left(\sqrt[3]{26+15\sqrt{3}}+\sqrt[3]{26-15\sqrt{3}}\right),$$

$$y_2=-\frac{1}{6}\left(\sqrt[3]{26+15\sqrt{3}}+\sqrt[3]{26-15\sqrt{3}}\right)+\frac{i\sqrt{3}}{6}\left(\sqrt[3]{26+15\sqrt{3}}-\sqrt[3]{26-15\sqrt{3}}\right),$$

$$y_3=-\frac{1}{6}\left(\sqrt[3]{26+15\sqrt{3}}+\sqrt[3]{26-15\sqrt{3}}\right)-\frac{i\sqrt{3}}{6}\left(\sqrt[3]{26+15\sqrt{3}}-\sqrt[3]{26-15\sqrt{3}}\right).$$

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