The analysis of an isolated rod's 4-momentum changes under the framework of Special Relativity

Abstract: Simplify an isolated rotating rod as an ideal system without any relative movement between the components, and the distribution of material and potential energy on the rod along the length is symmetry about the mid point and does not change with time, in an inertial reference frame. The 4 momentum of this isolated rod conservation in the inertial reference frame. Using Lorentz coordinate transformation formula and Lorentz velocity transformation formula between two inertial reference systems, find its 4-momentum does not equal to each other at different times in another inertial reference frame.
Keywords: Lorenz transformation; 4-momentum; Special Relativity
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In Special relativity, the motion of an object in non inertial motion can be described in an inertial reference frame and the motion of the object can be converted to be described in another inertial reference system, by Lorenz transformation. Rotating wheel has been extensively studied. Such as, measured in the inertial reference frame K, a wheel rotates around the mid point at uniform angular velocity, keeps its spokes as the linear state. The shape of the spokes will bent measured in another inertial reference frame K' [1][2][3].

Simplify one spokes on a rotating wheel as an ideal system without any relative movement between the components relative to K. 4-momentum of the spokes measured in K is conservation. Using Lorentz coordinate transformation formula and Lorentz velocity transformation formula between two inertial reference frames K and K' to calculate the 4 momentum of the spokes relative to K', the 4-momentum of the spokes on x' direction measured in K' is non-conservation. This is a result that is contrary to the current view.

1. Research object, premise

In K, an isolated rod *AB* rotates around its mid point *O* at uniform angular velocity ω, keeps the linear state. For each point on *AB*, the distance from point *O* to this point measured in K is constant; denote it is r. Such as for point *D*, r_D is a constant. For each point on *OA*, its equation of motion is $x = r\cos(\omega t)$, $y = r\sin(\omega t)$. For each point on *OB*, its equation of motion is $x = r\cos(\omega t+\pi)$, $y = r\sin(\omega t+\pi)$.

For actual rod *AB*, there is micro motion on molecular level. In the process of simplifying and establishing a mathematical model, we do not consider this micro motion. The rod *AB* is an ideal system without any relative movement between the components relative to K. The distribution of material and potential energy on the rod *AB* along the length is symmetry about mid point *O* and it does not change with time, measured in K. Rod AB is a Born rigid body. Rod *AB* has no thickness along θ direction, if transform K from rectangular coordinate system to polar coordinate system. That is to say consider rod *AB* as a one-dimensional rod on $x-y$ plane relative to K.

We consider: A body is static in an inertial reference system. When the body is free, it has a rest mass. When the body is stretched, it has another rest mass. The rest mass of the static body when it is stretched is greater than the rest mass of the body when it is free. When the body is static at different states such as free or stretched, the potential energy between the particles is different.

2. The relationship of the rest mass line density along *AB* between K and K'

In K, static auxiliary circle *D* and *M*, take *O* as their center of their circle, intersecting rod *OA* at point *D* and *M*, intersecting rod *OB* at point *E* and *N*. Here r_D and r_M can take any value from 0 to r_A.

In K, at time t, for each position on *OA*, denote $\rho_\alpha(t,r)$ as the rest mass line density at the position, thereinto

r is the distance from point *O* to this position. $\rho_\alpha(t,r)$ is equal to the rest mass of the segment at position r on *OA* divided by the length of this segment (the length of this segment need to be infinitesimal) at time t in K. $\rho_\alpha(t,r)$ is a function only relative to r, $\rho_\alpha(t,r)=\rho(r)$. Denote $\rho_\beta(t,r)$ as the rest mass line density for each position on *OB*. Because the mass distribution of *AB* is symmetry about mid point *O*, $\rho_\beta(t,r)=\rho_\alpha(t,r)=\rho(r)$.

The function's subscript α indicates that the function is valid for the point on *OA*. The function's subscript β indicates that the function is valid for the point on *OB*.

The inertial reference frame K' moves relative to K at speed v.

Relative to K', the auxiliary circle *D* and *M* appear as ellipses *D* and *M*, and they move at speed $-v$.

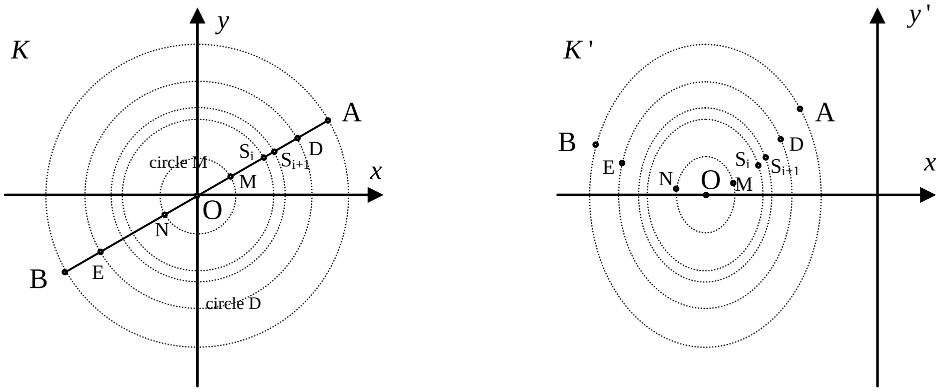

Fig 1 schematic diagram of *AB* and the auxiliary circle *D*, *M* in K and in K'. Section *MD* is marked into numerous segments. The length of each segment is infinitesimal. $r_{S_{i+1}}-r_{S_i}=\varepsilon$

Relative to K', rod *AB* can't keep the linear state at all times. In K', at time t', for each position on *AB*, use l' to represent the curve length from point *O* to this position along *AB*.

In K', at time t', for each point on *OA*, denote l' as the curve length from point *O* to this point along curve *OA*. It depends on the time t' and the position of this point on *OA*. Such as $l_D(t')'$ is the curve length from point *O* to point *D* along *AB* at time t' in K'. Because r represents the position of a point on *OA*, and r is a constant for each confirmed point, so for each point on *OA*, l' can be considered as a function of t' and r, remember it as $l'=f_\alpha(t',r)'$. For example, at the moment t_1', the length of the curve *OD* is $l_D(t_1')'=f_\alpha(t',r)'$, here the parameter $t'=t_1'$, $r=r_D$ (r_D is a constant).

In K', at time t', for each position on *OA*, denote $\rho_\alpha(t',l')'$ as the rest mass line density at the position when the curve length from point *O* to this position is l' measured in K' at the time t'. $\rho_\alpha(t',l')'$ is equal to the rest mass of the segment at position l' on *OA* divided by the length of this segment (the length of this segment need to be infinitesimal too) at the time t' in K'. $\rho_\alpha(t',l')'$ takes t' and l' as its parameters, t' is the time on K' and $l'=f_\alpha(t',r)'$ is the curve length from point *O* to this position along curve *OA* measured in K' at the time t'. Then $\rho_\alpha(t',l')'=\rho_\alpha(t',f_\alpha(t',r)')'$. So relative to K', for each position on *OA*, the rest mass line density also can be considered as it takes the time t' on K' and the distance r from point O to this position measured in K as parameters. Remember $g_\alpha(t',r)' = \rho_\alpha(t', f_\alpha(t',r)')'$.

In K', at time t', for each point on *OB*, denote l' as the curve length from point O to this point along curve *OB*, $l' = f_\beta(t',r)'$. For example, at the moment t_1', the length of the curve *OE* is $l_E(t_1')' = f_\beta(t',r)'$, here the parameter $t' = t_1'$, $r = r_E$.

In K', at time t', for each position on *OB*, denote $\rho_\beta(t',l')'$ as the rest mass line density at the position when the curve length from point O to this position is l' measured in K' at the time t'. $l' = f_\beta(t',r)'$, then $\rho_\beta(t',l')' = \rho_\beta(t', f_\beta(t',r)')'$. So relative to K', for each position on *OB*, the rest mass line density also can be considered as it takes the time t' on K' and the distance r from point O to this position measured in K as parameters. Remember $g_\beta(t',r)' = \rho_\beta(t', f_\beta(t',r)')'$.

Use $m_{sum\,0\,MD}(t)$ to represent the sum of rest masses of section *MD* at time t in K; and $m_{0\,MD}(t)$ to represent the invariant mass of section *MD* at time t in K. Here $m_{sum\,0\,MD}(t)$ isn't $m_{0\,MD}(t)$.

Use $m_{sum\,0\,MD}(t')'$ to represent the sum of rest masses of section *MD* at the time t' in K'.

$$m_{sum\,0\,MD}(t) = \int_{r_M}^{r_D} \rho_\alpha(t,r)dr = \int_{r_M}^{r_D} \rho(r)dr .$$

In K', at time t', the curve length of *OD* is $l_D(t')' = f_\alpha(t',r_D)'$. In K', the sum of rest masses of section *OD* is $m_{sum\,0\,OD}(t')' = \int_0^{l_D(t')'} \rho_\alpha(t',l')'dl'$. l' is a function of t' and r, $l' = f_\alpha(t',r)'$. Make variable substitution for the integral $l' \to r$,

$$m_{sum\,0\,OD}(t')' = \int_0^{r_D} \rho_\alpha(t', f_\alpha(t',r)')' \frac{\partial f_\alpha(t',r)'}{\partial r} dr = \int_0^{r_D} g_\alpha(t',r)' \frac{\partial f_\alpha(t',r)'}{\partial r} dr .$$

The same $m_{sum\,0\,OE}(t')' = \int_0^{r_E} g_\beta(t',r)' \frac{\partial f_\beta(t',r)'}{\partial r} dr$.

Rod AB is a Born rigid body, so $m_{sum\,0\,OD}(t) = m_{sum\,0\,OD}(t')'$.

$\int_0^{r_D} \rho(r)dr = \int_0^{r_D} g_\alpha(t',r)' \frac{\partial f_\alpha(t',r)'}{\partial r} dr$, and because of the arbitrariness of the auxiliary circle *D*, there is $g_\alpha(t',r)' \frac{\partial f_\alpha(t',r)'}{\partial r} = \rho(r)$ for any position on *OA* direction.

The same, there is $g_\beta(t',r)' \frac{\partial f_\beta(t',r)'}{\partial r} = \rho(r)$ for any position on *OB* direction.

3. Velocity character of each position on *AB* in K'

In K, at time t, the coordinates of point *D* on *OA* is $(r_D \cos(\omega t), r_D \sin(\omega t), 0, t)$; the coordinates of point *E* on

OB is $(r_E \cos(\omega t+\pi), r_E \sin(\omega t+\pi), 0, t)$; the velocity of point D is $u_\alpha(t, r_D)$; the velocity of point E is $u_\beta(t, r_E)$.

In K', at time t', for each point on OA, its velocity is $u_\alpha(t', l')'$, thereinto $l' = f_\alpha(t', r)'$; for each point on OB, its velocity is $u_\beta(t', l')'$, thereinto $l' = f_\beta(t', r)'$. l' represent the curve length from point O to this point along curve AB, such for position D, its velocity $u_\alpha(t', l_D(t')')'$ is $u_\alpha(t', f_\alpha(t', r_D)')'$

Set time $t_1'=0$ in K'.

In K', at time t_1', the event $E_{D\,t1'}$ occurs at the coordinates of point $D(x_{D1}', y_{D1}', 0, t_1')$. Using Lorentz transformation between K and K', translate coordinates $(x_{D1}', y_{D1}', 0, t_1')$ into the coordinates in K. The event $E_{D\,t1'}$ occurs at coordinates $(x_{D\,t_{D\,one}}, y_{D\,t_{D\,one}}, 0, t_{D\,one})$, or $(r_D \cos(\omega t_{D\,one}), r_D \sin(\omega t_{D\,one}), 0, t_{D\,one})$, the coordinates of point D at time $t_{D\,one}$ in K. $t_{D\,one} = t_\alpha(t_1', r_D)$.

In K', at time t_1', the event $E_{E\,t1'}$ occurs at the coordinates of point $E(x_{E1}', y_{E1}', 0, t_1')$. The event $E_{E\,t1'}$ occurs at $(r_E \cos(\omega t_{E\,one}+\pi), r_E \sin(\omega t_{E\,one}+\pi), 0, t_{E\,one})$, the coordinates of point E at time $t_{E\,one}$ in K.

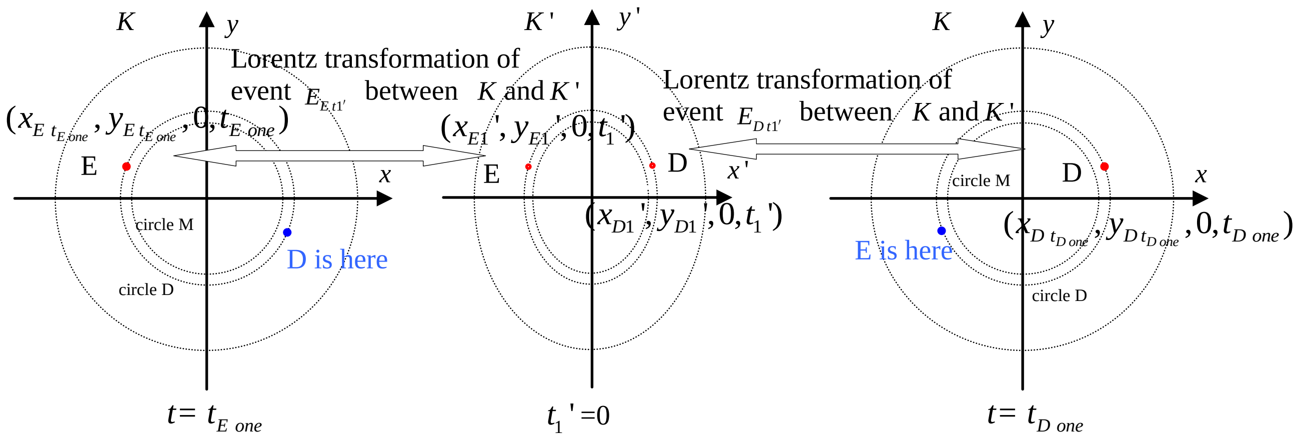

Fig 2 the relationship of t_1' and $t_{D\,one}$, $t_{E\,one}$.

Set time $t_{three} = \dfrac{\pi}{2\omega}$ in K.

In K, at time t_{three}, the event $E_{E\,t\,three}$ occurs at the coordinates of point E $(x_{Ethree}, y_{Ethree}, 0, t_{three})$, here $x_{Ethree}=0$. The event $E_{E\,t\,three}$ occurs at the coordinates of point E at time $t_3' = \gamma(v)\, t_{three} = \gamma(v)\dfrac{\pi}{2\omega}$ in K'.

In K, at time t_{three}, the event $E_{D\,t\,three}$ occurs at the coordinates of point D $(x_{Dthree}, y_{Dthree}, 0, t_{three})$, here $x_{Dthree}=0$. The event $E_{D\,t\,three}$ occurs at the coordinates of point D at time t_3' in K' too.

In K , at time $t_{D\,one}$, the coordinates of point D is $(r_D\cos(\omega t_{D\,one}), r_D\sin(\omega t_{D\,one}), 0, t_{D\,one})$, the velocity of point D is $u_\alpha(t_{D\,one}, r_D)$. In K', at time t_1', the velocity of point D is $u_D(t_1')' = u_\alpha(t_1', l_D(t_1')')'$.

$$u_{Dx}(t_{D\,one}) = u_{\alpha x}(t_{D\,one}, r_D) = -r_D\omega\sin(\omega t_{D\,one}) .$$

$$u_{Dy}(t_{D\,one}) = u_{\alpha y}(t_{D\,one}, r_D) = r_D\omega\cos(\omega t_{D\,one}) .$$

$$u_{Dx}(t_1')' = u_{\alpha x}(t_1', l_D(t_1')')' = \frac{u_{Dx}(t_{D\,one}) - v}{1 - \frac{v}{c^2}u_{Dx}(t_{D\,one})} = \frac{-r_D\omega\sin(\omega t_{D\,one}) - v}{1 + \frac{v}{c^2}r_D\omega\sin(\omega t_{D\,one})} .$$

$$u_{Dy}(t_1')' = u_{\alpha y}(t_1', l_D(t_1')')' = \frac{u_{Dy}(t_{D\,one})}{1 - \frac{v}{c^2}u_{Dx}(t_{D\,one})}\frac{1}{\gamma(v)} = \frac{r_D\omega\cos(\omega t_{D\,one})}{1 + \frac{v}{c^2}r_D\omega\sin(\omega t_{D\,one})}\sqrt{1 - \frac{v^2}{c^2}} .$$

$$\gamma(u_D(t_1')') = \gamma(u_\alpha(t_1', l_D(t_1')')') = 1/\sqrt{1 - \frac{u_D(t_1')'^2}{c^2}} = 1/\sqrt{1 - \frac{u_{Dx}(t_1')'^2 + u_{Dy}(t_1')'^2}{c^2}}$$

$$= (1 + \frac{v}{c^2}r_D\omega\sin(\omega t_{D\,one}))\,\gamma(v)\,\gamma(r_D\omega) .$$

$$\gamma(u_\alpha(t_1', l_D(t_1')')')\,u_{\alpha x}(t_1', l_D(t_1')')' = (-r_D\omega\sin(\omega t_{D\,one}) - v)\,\gamma(v)\,\gamma(r_D\omega) .$$

Set $\omega > 0, c > v = \omega r_A > 0$.

In K', at time t', for any point on *OA*, its coordinates is $(x', y', 0, t')$. Assume an event $E_{r\,t'}$ occurs at $(x', y', 0, t')$. Using Lorentz transformation between K and K', translate $(x', y', 0, t')$ into $(x, y, 0, t)$, here, t' is a function of t and r . Also it can be considered $t = t_\alpha(t', r)$. the coordinates of event $E_{r\,t'}$ in K . The event $E_{r\,t'}$ occurs at $(x, y, 0, t)$ in K . Here are $\gamma(u_\alpha(t', l')')\,u_{\alpha x}(t', l')'$ $= (-r\omega\sin(\omega t_\alpha(t', r)) - v)\,\gamma(v)\,\gamma(r\omega)$, $l' = f_\alpha(t', r)'$. Though r is a constant of each point. But for determined t', each r corresponds to l' one by one. So r can be considered as a function of t' and l'. Corresponding to the time $t' = t_1' = 0$, in K', for any point on *OA* except *O*, there are $\sin(\omega t_\alpha(t_1', r)) > 0$ (reason 1); Corresponding to the time $t' = t_3' = \gamma(v)\,t_{three}$, in K', for any point on *OA* except *O*, there are $t_\alpha(t_3', r) = t_{three} = \frac{\pi}{2\omega}$, $\sin(\omega t_\alpha(t_3', r)) = 1$.

In K', at time t', for any point on *OB*, its coordinates is $(x', y', 0, t')$. Event $E_{r\,t'}$ occurs at $(x', y', 0, t')$. Translate $(x', y', 0, t')$ into $(x, y, 0, t)$, the coordinates of event $E_{r\,t'}$ in K , here $t = t_\beta(t', r)$. The event $E_{r\,t'}$ occurs at $(x, y, 0, t)$ in K . Here are $\gamma(u_\beta(t', l')')\,u_{\beta x}(t', l')'$ $= (-r\omega\sin(\omega t_\beta(t', r) + \pi) - v)\,\gamma(v)\,\gamma(r\omega)$,

$l' = f_{\beta}(t',r)'$. Though r is a constant of each point. But for determined t', each r corresponds to l' one by one. So r can be considered as a function of t' and l'. Corresponding to the time $t' = t_1' = 0$, in K', for any point on *OB* except *O*, there are $\sin(\omega t_{\beta}(t_1',r)+\pi) > 0$ (reason 2); Corresponding to the time $t' = t_3' = \gamma(v)\, t_{three}$, in K', for any point on *OB* except *O*, there are $t_{\beta}(t_3',r) = t_{three} = \dfrac{\pi}{2\omega}$, $\sin(\omega t_{\beta}(t_3',r)+\pi) = -1$.

4. 4-momentum of *AB* in K' at time $t_1' = 0$ and $t_3' = \gamma(v)\, t_{three}$

For confirmed auxiliary circle *D* and auxiliary circle *M*, $r_D = r_E$, $r_M = r_N$. Now calculate 4-momentum of *MD* and *NE* on x' direction in K' at time $t_1' = 0$ and $t_3' = \gamma(v)\, t_{three}$.

$$P_{MDx}(t_1')' = \int_{l_M(t_1')'}^{l_D(t_1')'} \gamma(u_{\alpha}(t_1',l')')\, \rho_{\alpha}(t_1',l')'\, u_{\alpha x}(t_1',l')'dl' \qquad \text{(here } l' = f_{\alpha}(t_1',r)'.\ )$$

$$= \int_{l_M(t_1')'}^{l_D(t_1')'} \rho_{\alpha}(t_1',l')'(-r\omega\sin(\omega t_{\alpha}(t_1',r)) - v)\, \gamma(v)\, \gamma(r\omega) dl'$$

Though r is a constant of each point. But for determined $t' = t_1'$, each r corresponds to l' one by one. So r can be considered as a function of t_1' and l'. And l' also can be considered as a function of t_1' and r.

Because $\sin(\omega t_{\alpha}(t_1',r)) > 0$

$$P_{MDx}(t_1')' < \int_{l_M'}^{l_D'} -\rho_{\alpha}(t_1',l')'\, v\, \gamma(v)\, \gamma(r\omega) dl'$$

l' is function of t_1' and r, $l' = f_{\alpha}(t_1',r)'$. $t_1' = 0$ is a constant. Make variable substitution $l' \to r$

$$P_{MDx}(t_1')' < \int_{r_M}^{r_D} -\rho_{\alpha}(t_1', f_{\alpha}(t_1',r)')'\, \frac{\partial f_{\alpha}(t_1',r)'}{\partial r}\, v\, \gamma(v)\, \gamma(r\omega) dr$$

$$= \int_{r_M}^{r_D} -g_{\alpha}(t_1',r)'\, \frac{\partial f_{\alpha}(t_1',r)'}{\partial r}\, v\, \gamma(v)\, \gamma(r\omega) dr$$

$$= \int_{r_M}^{r_D} -\rho(r)\, v\, \gamma(v)\, \gamma(r\omega) dr\ .$$

$$P_{NEx}(t_1')' < \int_{r_N}^{r_E} -\rho(r)\, v\, \gamma(v)\, \gamma(r\omega) dr\ .$$

$$P_{MDx}(t_3')' = \int_{r_M}^{r_D} \rho(r)\, (-r\omega - v)\, v\, \gamma(v)\, \gamma(r\omega) dr\ .$$

$$P_{NEx}(t_3')' = \int_{r_N}^{r_E} \rho(r)\, (r\omega - v)\, v\, \gamma(v)\, \gamma(r\omega) dr\ .$$

For confirmed auxiliary circle *D* and auxiliary circle *M*, $r_D = r_E$, $r_M = r_N$.

$$\int_{r_M}^{r_D} -\rho(r)\, r\omega\, \gamma(v)\, \gamma(r\omega) dr + \int_{r_N}^{r_E} \rho(r)\, r\omega\, \gamma(v)\, \gamma(r\omega) dr = 0..$$

So $P_{MDx}(t_3')' + P_{NEx}(t_3')' = \int_{r_M}^{r_D} -\rho(r)\, v\, \gamma(v)\, \gamma(r\omega) dr + \int_{r_N}^{r_E} -\rho(r)\, v\, \gamma(v)\, \gamma(r\omega) dr$.

Then $P_{MDx}(t_1')' + P_{NEx}(t_1')' < P_{MDx}(t_3')' + P_{NEx}(t_3')'$.

When r_D =0, $r_M = r_A$, get the sum of the momentum of rod *AB* on x' direction along the curve *AB*, in K' at time t_1' and t_3':

$P_{BAx}(t_1')' < P_{BAx}(t_3')'$, under the condition $\omega > 0, c > v = \omega r_A > 0$.

The 4-momentum on x' direction of one isolated spokes measured in K' is non-conservation.

In K, *AB* is an isolated system, 4 momentum conservation. In K', the 4 momentum of *AB* must be conserved.

5. The momentum change is not caused by space-time bending.

Remember $k_{AB} = \frac{P_{BAx}(t_1')' - P_{BAx}(t_3')'}{P_{BAx}(t_3')'}$. Reduce the mass of the rod *AB* to half of the original mass, and then reduce it to half of the mass just now, until infinitesimal, in K. And the velocity of K' relative to K is not infinitely close to c ($\frac{v}{c}$ is not infinitely close to 1). Value of k_{AB} will not change at this process of the quality of rod *AB* changing. So k_{AB} is determined by parameters v, ω and r_A, and the result in paper is independent of the space-time bending.

6. Conclusion

The momentum of an isolated system consisting of particles is conserved when calculate the momentum at the times when there is no any interact between each other under any inertial reference frame. Of course at other intervals, particles can interact with each other. There is no doubt about it.

But now there is a result: The 4-momentum on x' direction of one isolated spokes is non-conservation, in K'.

To eliminate the contradictory result in paper, it is necessary that the potential energy of rod AB at t_1' is less than it at t_3', that is $m_{sum\,0\,AB}(t_1')' < m_{sum\,0\,AB}(t_3')'$, $m_{sum\,0\,AB}(t) \neq m_{sum\,0\,AB}(t')'$.

So the contradiction is that the internal potential energy of a rigid body varies relative to a fixed frame of reference, according to the requirements of special relativity. But rod AB is a born rigid body, its potential energy is unchanged, which has nothing to do with reference frame.

7. Thanks

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[3] Appearance of a Relativistically Rotating Disk, Keith MeFarlane, International Journal of Theoretical Physics, Vol.20. No.6, 1981.

reason 1:

$$\begin{cases} t_1' = \gamma(v)(t - \frac{v}{c^2}x) \\ x = r\cos(\omega t) \\ t = t_\alpha(t_1', r) \\ t_1' = 0 \end{cases}$$

$\omega t_\alpha(t_1', r) - \frac{v}{c^2} r\omega\cos(\omega t_\alpha(t_1', r))$ =0.

For any point on *OA* except *O*, 0< $r \le r_A$, $0 < r\omega < c$, 0< $\frac{v}{c^2} r\omega$ <1.(Because $0 < \omega, 0 < v$)

There is only one solution $0 < x < \pi/2$ satisfying for the equation $x - a\cos x = 0$, at condition 0< a <1.

So, 0< $\omega t_\alpha(t_1', r) < \pi/2$, $\sin(\omega t_\alpha(t_1', r))$ >0.

Reason 2: $t_1' = \gamma(v)(t_\beta(t_1', r) - \frac{v}{c^2} r\cos(\omega t_\beta(t_1', r) + \pi))$ =0,

$\omega t_\beta(t_1', r) - \frac{v}{c^2} r\omega\cos(\omega t_\beta(t_1', r) + \pi)$ =0,

$\omega t_\beta(t_1', r) + \frac{v}{c^2} r\omega\cos(\omega t_\beta(t_1', r))$ =0.

For any point on *OB* except *O*, $r \le r_B$, $0 < r\omega < c$, 0< $\frac{v}{c^2} r\omega$ <1. (Because $0 < \omega, 0 < v$)

There is only one solution $-\pi/2 < x$ <0 satisfying for the equation $x + a\cos x = 0$, at condition 0< a <1.

So, $-\pi/2 < \omega t_\beta(t_1', r)$ <0, $\sin(\omega t_\beta(t_1', r) + \pi)$ >0.

Detailed process $1/\sqrt{1 - \frac{u_D(t_1')'^2}{c^2}}$ $=1/\sqrt{1 - \frac{u_{Dx}(t_1')'^2 + u_{Dy}(t_1')'^2}{c^2}}$

$$= \frac{1 + \frac{v}{c^2} r_D\omega\sin(\omega t_{D\,one})}{\sqrt{(1 + \frac{v}{c^2} r_D\omega\sin(\omega t_{D\,one}))^2 - \frac{(-r_D\omega\sin(\omega t_{D\,one}) - v)^2 + (r_D\omega\cos(\omega t_{D\,one}))^2(1 - \frac{v^2}{c^2})}{c^2}}}$$

$$= \frac{(1 + \frac{v}{c^2} r_D\omega\sin(\omega t_{D\,one}))c^2}{\sqrt{(c^2 + v r_D\omega\sin(\omega t_{D\,one}))^2 - c^2(-r_D\omega\sin(\omega t_{D\,one}) - v)^2 - (r_D\omega\cos(\omega t_{D\,one}))^2(c^2 - v^2)}}$$

$$= \frac{(1 + \frac{v}{c^2} r_D\omega\sin(\omega t_{D\,one}))c^2}{\sqrt{\begin{array}{l} c^4 + v^2 r_D{}^2\omega^2\sin(\omega t_{one})^2 + 2c^2 v r_D\omega\sin(\omega t_{D\,one}) - c^2 r_D{}^2\omega^2\sin(\omega t_{D\,one})^2 - c^2v^2 - 2c^2 r_D\omega v\sin(\omega t_{D\,one}) \\ -c^2 r_D{}^2\omega^2\cos(\omega t_{one})^2 + v^2 r_D{}^2\omega^2\cos(\omega t_{D\,one})^2 \end{array}}}$$

$$= \frac{(1 + \frac{v}{c^2} r_D\omega\sin(\omega t_{D\,one}))c^2}{\sqrt{c^4 + v^2 r_D{}^2\omega^2 - c^2 r_D{}^2\omega^2 - c^2v^2}} = \frac{(1 + \frac{v}{c^2} r_D\omega\sin(\omega t_{D\,one}))c^2}{\sqrt{c^4 - c^2 r_D{}^2\omega^2 - c^2v^2 + v^2 r_D{}^2\omega^2}}$$

$$=\frac{(1+\frac{v}{c^2}r_D\omega\sin(\omega t_{D\,one}))c^2}{\sqrt{(c^2-r_D{}^2\omega^2)(c^2-v^2)}}=(1+\frac{v}{c^2}r_D\omega\sin(\omega t_{D\,one}))\gamma(v)\,\gamma(r_D\omega)$$

Detailed process $P_{NEx}(t_1')'=\int_{l_N'}^{l_E'}\gamma(u_\beta(t_1',l')')\rho_\beta(t_1',l')'u_{\beta x}(t_1',l')'dl'$

$=\int_{l_N'}^{l_E'}\rho_\beta(t_1',l')'(-r\omega\sin(\omega t_\beta(t_1',r)+\pi)-v)\gamma(v)\,\gamma(r\omega)dl'$, because $\sin(\omega t_\beta(t_1',r)+\pi)>0$

$<\int_{l_N'}^{l_E'}-\rho_\beta(t_1',l')'v\gamma(v)\,\gamma(r\omega)dl'$

$$=\int_{r_N}^{r_E}-\rho_\beta(t_1',f_\beta(t_1',r)')'\frac{\partial f_\beta(t_1',r)'}{\partial r}v\gamma(v)\,\gamma(r\omega)dr$$

$$=\int_{r_N}^{r_E}-g_\beta(t_1',r)'\frac{\partial f_\beta(t_1',r)'}{\partial r}v\gamma(v)\,\gamma(r\omega)dr$$

$$=\int_{r_N}^{r_E}-\rho(r)v\gamma(v)\,\gamma(r\omega)dr$$

Detailed process $P_{MDx}(t_3')'=\int_{l_M'}^{l_D'}\gamma(u_\alpha(t_3',l')')\rho_\alpha(t_3',l')'u_{\alpha x}(t_3',l')'dl'$

$=\int_{l_M'}^{l_D'}\rho_\alpha(t_3',l')'(-r\omega\sin(\omega t_\alpha(t_3',r))-v)\gamma(v)\,\gamma(r\omega)dl'$, because $\sin(\omega t_\alpha(t_3',r))=1$

$$=\int_{l_M'}^{l_D'}\rho_\alpha(t_3',l')'(-r\omega-v)\gamma(v)\,\gamma(r\omega)dl'$$

$$=\int_{r_M}^{r_D}\rho_\alpha(t_3',f_\alpha(t_3',r)')'\frac{\partial f_\alpha(t_3',r)'}{\partial r}(-r\omega-v)\gamma(v)\,\gamma(r\omega)dr$$

$$=\int_{r_M}^{r_D}g_\alpha(t_3',r)'\frac{\partial f_\alpha(t_3',r)'}{\partial r}(-r\omega-v)\gamma(v)\,\gamma(r\omega)dr$$

$$=\int_{r_M}^{r_D}\rho(r)(-r\omega-v)\gamma(v)\,\gamma(r\omega)dr$$

Detailed process $P_{NEx}(t_3')'=\int_{l_N'}^{l_E'}\gamma(u_\beta(t_3',l')')\rho_\beta(t_3',l')'u_{\beta x}(t_3',l')'dl'$

$=\int_{l_N'}^{l_E'}\rho_\beta(t_3',l')'(-r\omega\sin(\omega t_\beta(t_3',r)+\pi)-v)\gamma(v)\,\gamma(r\omega)dl'$, because $\sin(\omega t_\beta(t_3',r)+\pi)=-1$

$$=\int_{l_N'}^{l_E'}\rho_\beta(t_3',l')'(r\omega-v)\gamma(v)\,\gamma(r\omega)dl'$$

$$=\int_{r_N}^{r_E}\rho_\beta(t_3',f_\beta(t_3',r)')'\frac{\partial f_\beta(t_3',r)'}{\partial r}(r\omega-v)\gamma(v)\,\gamma(r\omega)dr$$

$$=\int_{r_N}^{r_E}g_\beta(t_3',r)'\frac{\partial f_\beta(t_3',r)'}{\partial r}(r\omega-v)\gamma(v)\,\gamma(r\omega)dr$$

$$=\int_{r_N}^{r_E}\rho(r)(r\omega-v)\gamma(v)\,\gamma(r\omega)dr$$