

## Light Speed(s)

### 1. Simple definitions

Let  $\omega$  be the angular velocity and  $k$  the wavenumber<sup>1</sup>, then the phase velocity  $V_f$  is defined as

$$V_f = \omega/k \quad (1)$$

The group velocity is the velocity with which energy propagates and is defined by

$$V_g = \delta\omega/\delta k \quad (2)$$

### 2. The case of light

If a wave satisfies the dispersion relation then:

$$\omega(k) = \sqrt{k^2 c^2 + \text{const}} \quad (3)$$

Expression (3) is exactly the case for light.

From (1), (2) and (3) we obtain

$$V_g * V_f = c^2 \quad (4)$$

In (4)  $V_f$  or  $V_g$  can be greater than  $c$  or even negative. Though  $V_f$  or  $V_g$  may exceed  $c$  no energy or information actually travels faster than  $c$ . Experiments showing group velocities greater than  $c$  include that of Wang *et al.*<sup>2</sup>, who produced a laser pulse in atomic cesium gas with  $V_g$  between  $(-310+5)c$  and  $(-310-5)c$ .  $V_f$  is in the range  $-c/305$  to  $-c/315$ .

### 3. References

1. Feynman, Leighton and Sands (vol1) Addison-Wesley
2. Wang, L. J.; Kuzmich, A.; and Dogariu, A. "Gain-Assisted Superluminal Light Propagation." *Nature* **406**, 277-279, 2000.