

Cylinder, radius R , rotating counter-clockwise with angular velocity ω and tangential velocity ωR

→ `/* initial position */`

`kill(all);`

`x0:0;`

`y0:-R;`

`(%o0) done`

`(x0) 0`

`(y0) -R`

→ `ipos:[x0,y0];`

`(ipos) [0,-R]`

Initial velocity ($t=0$) magnitude is given as a percentage, f , of the tangential velocity: $f * \omega * R$

The velocity angle θ , measured counter-clockwise from x-axis, determines the vector components.

The velocity angle is the angle of throw from the initial position after a displacement to the origin $(0,0)$.

Angles (θ) are in the range $[-\pi, \pi]$.

IOW, determine the velocity direction at a point on the circumference, then dispace this direction to the origin

and give a value in $[-\pi, \pi]$, i.e. counter-clockwise from the positive x-axis.

→ `/* initial velocity as seen in the rotating frame */`

`ivel:[f*R*omega*cos(theta), f*R*omega*sin(theta)];`

`(ivel) [R f cos(theta) omega, R f sin(theta) omega]`

→ `tranmat:trigsimp(invert(matrix([cos(omega*t), -sin(omega*t)], [sin(omega*t), cos(omega*t)])));`

`(tranmat)`
$$\begin{pmatrix} \cos(t\omega) & \sin(t\omega) \\ -\sin(t\omega) & \cos(t\omega) \end{pmatrix}$$

→ `/* calculate the trajectory in the inertial frame by using the transformed velocity */`

`traj:ipos+ivel*t+omega*t*[-y0, x0];`

`(traj) [R f t cos(theta) omega + R t omega, R f t sin(theta) omega - R]`

→ `/* translate the trajectory back into the rotating frame */`

`rottraj:tranmat.(traj);`

`(rottraj)`
$$\begin{pmatrix} (R f t \sin(\theta) \omega - R) \sin(t\omega) + (R f t \cos(\theta) \omega + R t \omega) \cos(t\omega) \\ (R f t \sin(\theta) \omega - R) \cos(t\omega) - (R f t \cos(\theta) \omega + R t \omega) \sin(t\omega) \end{pmatrix}$$

→ `load("draw");`

(%o8) `/opt/maxima/share/maxima/5.43.0/share/draw/draw.lisp`

→ `set_draw_defaults(
 axis_3d=true,
 background_color=gray80,
 contour=none,
 grid=true,
 ip_grid=[50,50], ip_grid_in=[5,5],
 line_type=solid, line_width=1,
 nticks=200,
 point_type=0, point_size=1,
 points_joined=false,
 xaxis=true, yaxis=true, zaxis=true, xaxis_type=solid, yaxis_type=solid, zaxis_type=solid, axis_3d=
 xyplane=0, proportional_axes=none,
 xu_grid=50, yv_grid=50,
 surface_hide=true,
 enhanced3d=none);`

(%o9) `[axis_3d = true, background_color = gray80, contour = none, grid = true,
 ip_grid = [50, 50], ip_grid_in = [5, 5], line_type = solid, line_width = 1, nticks = 200
 , point_type = 0, point_size = 1, points_joined = false, xaxis = true, yaxis = true,
 zaxis = true, xaxis_type = solid, yaxis_type = solid, zaxis_type = solid, axis_3d =
 true, xyplane = 0, proportional_axes = none, xu_grid = 50, yv_grid = 50,
 surface_hide = true, enhanced3d = none]`

→ `define(u(ω ,f, θ ,R,t), first(list_matrix_entries(row(rotttraj,1))));
 define(v(ω ,f, θ ,R,t), first(list_matrix_entries(row(rotttraj,2))));`

(%o8) $u(\omega, f, \theta, R, t) := (R f t \sin(\theta) \omega - R) \sin(t \omega) +$
 $(R f t \cos(\theta) \omega + R t \omega) \cos(t \omega)$

(%o9) $v(\omega, f, \theta, R, t) := (R f t \sin(\theta) \omega - R) \cos(t \omega) -$
 $(R f t \cos(\theta) \omega + R t \omega) \sin(t \omega)$

```
→ ω:1; f:1; θ:%pi/2+1.23778; R:1;
wxdraw2d(xrange=[-1.2,1.2], yrange=[-1.2,1.2], color=red, parametric(R·cos(t), R·sin(t), t, 0, 2·%pi)
parametric(u(ω,f,θ,R,t), v(ω,f,θ,R,t), t, 0, 10·%pi))$;
```

(%o15) 1

(f) 1

(%o17) $\frac{\pi}{2} + 1.23778$

(R) 1

(%t19)

