



Data

$$m \in \mathbb{R}$$

$$\theta \in \mathbb{R}$$

$$v(0) = v_0$$

$$\vec{F}_D = -B|\vec{v}|$$

Solution

$$\sum \vec{F} = m \cdot \frac{\partial v(t)}{\partial t} \Leftrightarrow -B \cdot v(t) - m \cdot g \cdot \sin \theta = m \cdot \frac{\partial v(t)}{\partial t}$$

$$= m \cdot \frac{\partial v(t)}{\partial x} \cdot \frac{\partial x}{\partial t} = m \cdot \frac{\partial v(t)}{\partial x} \cdot v(t)$$

~~$$m \cdot \frac{\partial v(t)}{\partial x} \cdot v(t) = -m \cdot g \cdot \sin \theta - B \cdot v(t)$$~~

$$\Leftrightarrow m \cdot \frac{\partial v(t)}{\partial x} \cdot v(t) = [-m \cdot g \cdot \sin \theta - B \cdot v(t)] \partial x$$

$$\Leftrightarrow \partial x = \frac{m \cdot v(t)}{-m g \sin \theta - B v(t)} \partial v(t)$$

$$\Leftrightarrow \int_{x(0)}^{x(t)} \partial x = -m \int_0^t \frac{v(t)}{B v(t) + m g \sin \theta} \partial v(t)$$

$$\Rightarrow \boxed{x(t) - x(0) = -m \int_0^t \frac{v(t)}{Bv(t) + m \cdot g \cdot \sin \theta} dv(t)} \quad (1)$$

$$\begin{array}{r|l} v(t) & Bv(t) + m \cdot g \cdot \sin \theta \\ -\left(v(t) + \frac{m \cdot g \cdot \sin \theta}{B}\right) & \frac{1}{B} \\ \hline -\frac{m \cdot g \cdot \sin \theta}{B} & \end{array}$$

$$x(t) - x(0) = -m \int_0^t \frac{1}{B} - \frac{m \cdot g \cdot \sin \theta}{B^2 v(t) + B \cdot m \cdot g \cdot \sin \theta} dv(t)$$

$$= -m \left[\int_0^t \frac{1}{B} dv(t) - \frac{m \cdot g \cdot \sin \theta}{B^2} \int_0^t \frac{B^2 dv(t)}{B^2 v(t) + B \cdot m \cdot g \cdot \sin \theta} \right]$$

Let $u = B^2 v(t) + B \cdot m \cdot g \cdot \sin \theta$, $u(0) = B \cdot m \cdot g \cdot \sin \theta$

$$du = B^2 dv(t)$$

$$= -m \left[\frac{1}{B} [v(t)]_0^t - \frac{m \cdot g \cdot \sin \theta}{B^2} \int_{B \cdot m \cdot g \cdot \sin \theta}^{u(t)} \frac{du(t)}{u(t)} \right]$$

$$= -m \left[\frac{1}{B} \cdot t - \frac{m \cdot g \cdot \sin \theta}{B^2} \left[\ln(B^2 v(t) + B \cdot m \cdot g \cdot \sin \theta) - \ln(B \cdot m \cdot g \cdot \sin \theta) \right] \right]$$

$$= -m \left[\frac{1}{B} \cdot t - \frac{m \cdot g \cdot \sin \theta}{B^2} \cdot \ln \frac{B^2 v(t) + B \cdot m \cdot g \cdot \sin \theta}{B \cdot m \cdot g \cdot \sin \theta} \right]$$

$$x(t) = -m \left[\frac{t}{B} - \frac{m \cdot g \cdot \sin \theta}{B^2} \cdot \ln \left(\frac{B v(t) + m \cdot g \cdot \sin \theta}{m \cdot g \cdot \sin \theta} \right) \right] + x(0)$$

$$= -m \left[\frac{Bt - m \cdot g \cdot \sin \theta}{B^2} \cdot \ln \left(\frac{B \cdot v(t) + m \cdot g \cdot \sin \theta}{m \cdot g \cdot \sin \theta} \right) \right] + x(0)$$

$$x(t) = - \frac{m \cdot B \cdot t - m^2 \cdot g \cdot \sin \theta}{B^2} \cdot \ln \frac{B v(t) + m \cdot g \cdot \sin \theta}{m \cdot g \cdot \sin \theta} + x(0) \quad (2)$$