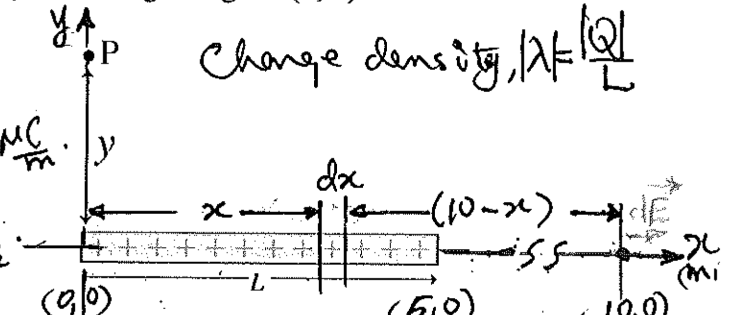
PHYS-2212 Due: ____________

Class Activity 02 Name: ____________

ELECTRIC FIELDS – CONTINUOUS CHARGE DISTRIBUTIONS (2)

1. A plastic rod of finite length carries a uniform linear charge $Q = 50\ \mu C$ along the x-axis, with the left edge of the rod at the origin (0, 0) and its right edge at (5, 0) m. All distances are measured in meters.

Charge density, $|\lambda| = \frac{|Q|}{L}$



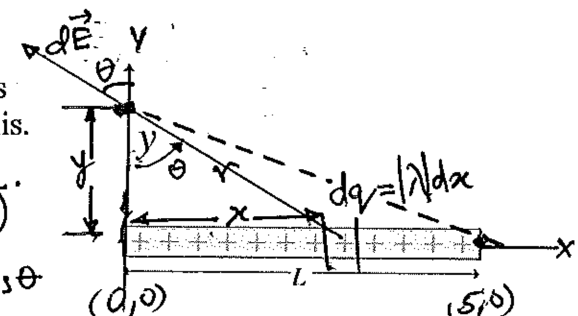
(a) Determine the net electric field at a point (10,0) m, along the positive x-axis.

$d\vec{E} = \hat{i}\,\frac{K|\lambda|dx}{(10-x)^2}$ and $|\lambda| = 10\,\frac{\mu C}{m}$.

$\therefore\ \vec{E} = \hat{i}\int \frac{K|\lambda|\,dx}{(10-x)^2} = \hat{i}K|\lambda|\int_0^5 \frac{dx}{(10-x)^2}$.

i.e. $\vec{E} = \hat{i}\left(90{,}000\ \frac{N\cdot m}{C}\right)\left[\frac{1}{10-x}\right]_0^{5m} \Rightarrow \vec{E} = \hat{i}\left(9\times10^3\ \frac{N}{C}\right)$.

(b) Apply integral methods to find the x- and y-components of the electric field vector due to this charged rod at the point P(0, 3) m, along the y-axis.



magnitude; $|d\vec{E}| = \frac{K|\lambda|dx}{r^2} = \frac{K|\lambda|\,dx}{(y^2+x^2)}$.

Components;
$dE_x = dE\sin\theta$ and $dE_y = dE\cos\theta$

Summing components:
$\vec{E}_x = -\hat{i}\int dE\sin\theta$ and $\vec{E}_y = \hat{j}\int dE\cos\theta$.

$\sin\theta = \frac{x}{r} = \frac{x}{\sqrt{y^2+x^2}}$.

$\cos\theta = \frac{y}{r} = \frac{y}{\sqrt{y^2+x^2}}$.

Simplifying:
$\vec{E}_x = -\hat{i}K|\lambda|\int_0^{L=5m} \frac{x}{(y^2+x^2)^{3/2}}dx = -\hat{i}(14{,}565\ N/C)$

and $\vec{E}_y = \hat{j}K|\lambda|\int_0^L \frac{y}{(y^2+x^2)^{3/2}}dx = \hat{j}(25{,}724\ N/C)$

or $\vec{E} = 29{,}561\ N/C$ @ 119.5°.

(c) If the right end of this charged rod were extended to an infinite distance along the positive x-axis, determine the magnitude and direction of the net electric field at point P(0,3) m.

When the right end extends to infinity along +ve x-axis; angle θ extends from 0° to 90°. $\therefore$ Changing variables.
Let $x = y\tan\theta$, so $dx = y\sec^2\theta\,d\theta$ and $(y^2+x^2) = y^2\sec^2\theta$.

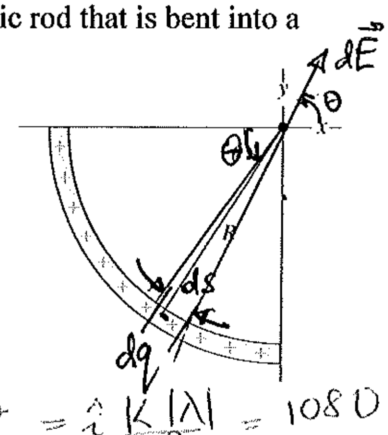
$\therefore\ dE = \frac{K|\lambda|\,dx}{(y^2+x^2)} = \frac{K|\lambda|}{y}d\theta$ So, components

$\vec{E}_x = -\hat{i}\int dE\sin\theta = -\hat{i}\frac{K|\lambda|}{y}\int_0^{90^\circ}\sin\theta\,d\theta = -\hat{i}\frac{K|\lambda|}{y}$

and $\vec{E}_y = \hat{j}\int dE\cos\theta = \hat{j}\frac{K|\lambda|}{y}\int_0^{90^\circ}\cos\theta\,d\theta = \hat{j}\frac{K|\lambda|}{y}$

$\therefore\ \vec{E} = 42{,}426\ N/C$ @ 135°

2. A linear charge density of 12 nC/m exists throughout a plastic rod that is bent into a quarter of a circle of radius 10 cm. Find the magnitude and direction of the electric field at the center of this circular shape, as shown.



$dq = |\lambda| ds = |\lambda| R d\theta$.

Since

$d\vec{E}_x = \hat{i}(dE\cos\theta) = \hat{i}\,\frac{K|\lambda|}{R}\cos\theta\,d\theta$

$d\vec{E}_y = \hat{j}(dE\sin\theta) = \hat{j}\,\frac{K|\lambda|}{R}\sin\theta\,d\theta$

integrating, $\vec{E}_x = \hat{i}\,\frac{K|\lambda|}{R}\int_0^{\pi/2}\cos\theta\,d\theta = \hat{i}\,\frac{K|\lambda|}{R} = 1080\ \text{N/C}\ \hat{i}$.

and $\vec{E}_y = \hat{j}\,\frac{K|\lambda|}{R}\int_0^{\pi/2}\sin\theta\,d\theta = \frac{K|\lambda|}{R} = 1080\ \frac{N}{C}\ \hat{j}$.

$\therefore$ Electric field, $\vec{E} = 1{,}527$ N/C @ 45°

Some useful integrals:

$$\int\frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}} \qquad \int\frac{x\,dx}{(x^2+a^2)^{3/2}} = -\frac{1}{(x^2+a^2)^{1/2}}$$

$$\int\frac{x^2}{\sqrt{(x^2+a^2)^3}} = \frac{-x}{\sqrt{x^2+a^2}} + \log\left(x+\sqrt{x^2+a^2}\right)$$