SOLUTION

EXAMPLE 38.3 DETERMINING ϕ AND h EXPERIMENTALLY

For a particular cathode material in a photoelectric-effect experiment, you measure stopping potentials $V_0 = 1.0$ V for light of wavelength $\lambda = 600$ nm, 2.0 V for 400 nm, and 3.0 V for 300 nm. Determine the work function ϕ for this material and the implied value of Planck's constant h.

SOLUTION

IDENTIFY and SET UP: This example uses the relationship among stopping potential V_0, frequency f, and work function ϕ in the photoelectric effect. According to Eq. (38.4), a graph of V_0 versus f should be a straight line as in Fig. 38.5 or 38.6. Such a graph is completely determined by its slope and the value at which it intercepts the vertical axis; we will use these to determine the values of the target variables ϕ and h.

EXECUTE: We rewrite Eq. (38.4) as

$$V_0 = \frac{h}{e}f - \frac{\phi}{e}$$

In this form we see that the slope of the line is h/e and the vertical-axis intercept (corresponding to $f = 0$) is $-\phi/e$. The frequencies, obtained from $f = c/\lambda$ and $c = 3.00 \times 10^8$ m/s, are 0.50×10^{15} Hz, 0.75×10^{15} Hz, and 1.0×10^{15} Hz, respectively. From a graph of these data (see Fig. 38.6), we find

$$-\frac{\phi}{e} = \text{vertical intercept} = -1.0\text{ V}$$

$$\phi = 1.0\text{ eV} = 1.6 \times 10^{-19}\text{ J}$$

and

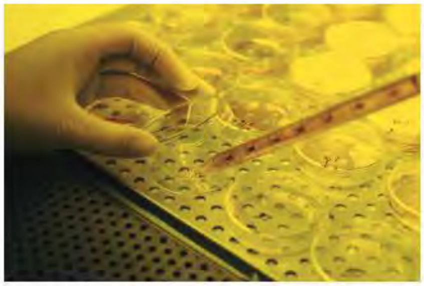
$$\text{Slope} = \frac{\Delta V_0}{\Delta f} - \frac{3.0\text{ V} - (-1.0\text{ V})}{1.00 \times 10^{15}\text{ s}^{-1} - 0} = 4.0 \times 10^{-15}\text{ J}\cdot\text{s/C}$$

$$h = \text{slope} \times e = (4.0 \times 10^{-15}\text{ J}\cdot\text{s/C})(1.60 \times 10^{-19}\text{ C})$$

$$= 6.4 \times 10^{-34}\text{ J}\cdot\text{s}$$

EVALUATE: The value of Planck's constant h determined from your experiment differs from the accepted value by only about 3%. The small value $\phi = 1.0$ eV tells us that the cathode surface is not composed solely of one of the elements in Table 38.1.

BIO Application Sterilizing with High-Energy Photons One technique for killing harmful microorganisms is to illuminate them with ultraviolet light with a wavelength shorter than 254 nm. If a photon of such short wavelength strikes a DNA molecule within a microorganism, the energy of the photon is great enough to break the bonds within the molecule. This renders the microorganism unable to grow or reproduce. Such ultraviolet germicidal irradiation is used for medical sanitation, to keep laboratories sterile (as shown here), and to treat both drinking water and wastewater.



TEST YOUR UNDERSTANDING OF SECTION 38.1 Silicon films become better electrical conductors when illuminated by photons with energies of 1.14 eV or greater, an effect called *photoconductivity*. Which of the following wavelengths of electromagnetic radiation can cause photoconductivity in silicon films? (i) Ultraviolet light with $\lambda = 300$ nm; (ii) red light with $\lambda = 600$ nm; (iii) infrared light with $\lambda = 1200$ nm; (iv) both (i) and (ii); (v) all of (i), (ii), and (iii). ❙

38.2 LIGHT EMITTED AS PHOTONS: X-RAY PRODUCTION

The photoelectric effect provides convincing evidence that light is *absorbed* in the form of photons. For physicists to accept Einstein's radical photon concept, however, it was also necessary to show that light is *emitted* as photons. An experiment that demonstrates this convincingly is the inverse of the photoelectric effect: Instead of releasing electrons from a surface by shining electromagnetic radiation on it, we cause a surface to emit radiation—specifically, *x rays*—by bombarding it with fast-moving electrons.

X-Ray Photons

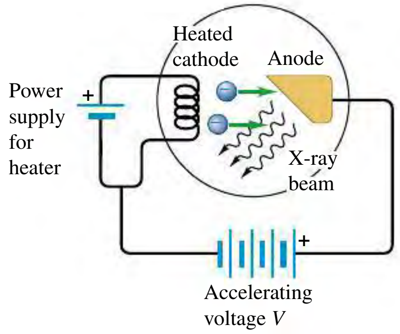
X rays were first produced in 1895 by the German physicist Wilhelm Röntgen, using an apparatus similar in principle to the setup shown in **Fig. 38.7**. When the cathode is heated to a very high temperature, it releases electrons in a process called *thermionic emission*. (As in the photoelectric effect, the minimum energy that an individual electron must be given to escape from the cathode's surface is equal to the work function for the surface. In this case the energy is provided to the electrons by heat rather than by light.) The electrons are then accelerated toward the anode by a potential difference V_{AC}. The bulb is evacuated (residual pressure 10^{-7} atm or less), so the electrons can travel from the cathode to the anode without colliding with air molecules. When V_{AC} is a few thousand volts or more, x rays are emitted from the anode surface.

The anode produces x rays in part simply by slowing the electrons abruptly. (Recall from Section 32.1 that accelerated charges emit electromagnetic waves.) This process is called *bremsstrahlung* (German for "braking radiation"). Because the electrons undergo accelerations of very great magnitude, they emit much of their radiation at short wavelengths in the x-ray range, about 10^{-9} to 10^{-12} m (1 nm to 1 pm). (X-ray wavelengths can be measured quite precisely by crystal diffraction techniques, which we discussed in Section 36.6.) Most electrons are braked by a series of collisions and interactions with anode atoms, so bremsstrahlung produces a continuous spectrum of electromagnetic radiation.

Just as we did for the photoelectric effect in Section 38.1, let's compare what Maxwell's wave theory of electromagnetic radiation would predict about this radiation to what is observed experimentally.

38.7 An apparatus used to produce x rays, similar to Röntgen's 1895 apparatus.

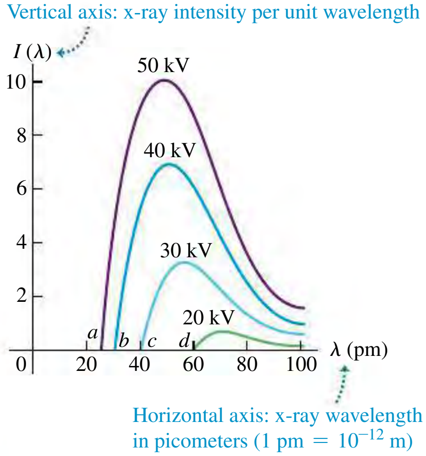
Electrons are emitted thermionically from the heated cathode and are accelerated toward the anode; when they strike it, x rays are produced.



Wave-Model Prediction: The electromagnetic waves produced when an electron slams into the anode should be analogous to the sound waves produced by crashing cymbals together. These waves include sounds of all frequencies. By analogy, the x rays produced by bremsstrahlung should have a spectrum that includes *all* frequencies and hence *all* wavelengths.

Experimental Result: **Figure 38.8** shows bremsstrahlung spectra obtained when the same cathode and anode are used with four different accelerating voltages V_{AC}. *Not* all x-ray frequencies and wavelengths are emitted: Each spectrum has a maximum frequency f_{max} and a corresponding minimum wavelength λ_{min}. The greater the value of V_{AC}, the higher the maximum frequency and the shorter the minimum wavelength.

38.8 The continuous spectrum of x rays produced when a tungsten target is struck by electrons accelerated through a voltage V_{AC}. The curves represent different values of V_{AC}; points a, b, c, and d show the minimum wavelength for each voltage.



The wave model of electromagnetic radiation cannot explain these experimental results. But we can readily understand them by using the photon model. An electron has charge $-e$ and gains kinetic energy eV_{AC} when accelerated through a potential increase V_{AC}. The most energetic photon (highest frequency and shortest wavelength) is produced if the electron is braked to a stop all at once when it hits the anode, so that all of its kinetic energy goes to produce one photon; that is,

Bremsstrahlung:

$$eV_{AC} = hf_{max} = \frac{hc}{\lambda_{min}} \quad (38.6)$$

Kinetic energy lost by electron (eV_{AC}); Maximum energy of an emitted photon (hf_{max}); Planck's constant (h); Speed of light in vacuum (c); Magnitude of electron charge (e); Accelerating voltage (V_{AC}); Maximum photon frequency (f_{max}); Minimum photon wavelength (λ_{min})

(In this equation we ignore the work function of the target anode and the initial kinetic energy of the electrons "boiled off" from the cathode. These energies are very small compared to the kinetic energy eV_{AC} gained due to the potential difference.) If only a portion of an electron's kinetic energy goes into producing a photon, the photon energy will be less than eV_{AC} and the wavelength will be greater than λ_{min}. Experiment shows that the measured values for λ_{min} for different values of eV_{AC} (see Fig. 38.8) agree with Eq. (38.6). Note that according to Eq. (38.6), the maximum frequency and minimum wavelength in the bremsstrahlung process do not depend on the target material; this also agrees with experiment. So we can conclude that the photon picture of electromagnetic radiation is valid for the *emission* as well as the absorption of radiation.

The apparatus shown in Fig. 38.7 can also produce x rays by a second process in which electrons transfer their kinetic energy partly or completely to individual atoms within the target. It turns out that this process not only is consistent with the photon model of electromagnetic radiation, but also provides insight into the structure of atoms. We'll return to this process in Section 41.5.

SOLUTION

EXAMPLE 38.4 PRODUCING X RAYS

Electrons in an x-ray tube accelerate through a potential difference of 10.0 kV before striking a target. If an electron produces one photon on impact with the target, what is the minimum wavelength of the resulting x rays? Find the answer by expressing energies in both SI units and electron volts.

SOLUTION

IDENTIFY and SET UP: To produce an x-ray photon with minimum wavelength and hence maximum energy, all of the electron's kinetic energy must go into producing a single x-ray photon. We'll use Eq. (38.6) to determine the wavelength.

EXECUTE: From Eq. (38.6), using SI units we have

$$\lambda_{\min} = \frac{hc}{eV_{AC}} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{(1.602 \times 10^{-19}\ \text{C})(10.0 \times 10^{3}\ \text{V})}$$
$$= 1.24 \times 10^{-10}\ \text{m} = 0.124\ \text{nm}$$

Using electron volts, we have

$$\lambda_{\min} = \frac{hc}{eV_{AC}} = \frac{(4.136 \times 10^{-15}\ \text{eV}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{e(10.0 \times 10^{3}\ \text{V})}$$
$$= 1.24 \times 10^{-10}\ \text{m} = 0.124\ \text{nm}$$

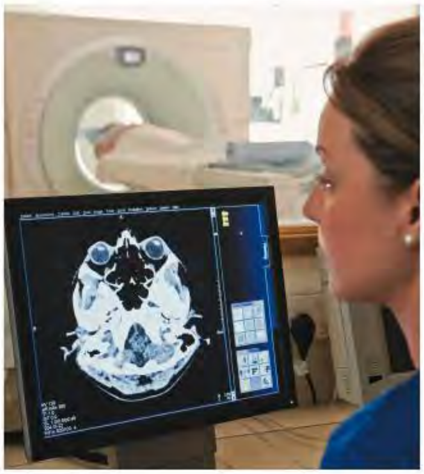
In the second calculation, the "e" for the magnitude of the electron charge cancels the "e" in the unit "eV," because the electron volt (eV) is the magnitude of the electron charge e times one volt (1 V).

EVALUATE: To check our result, recall from Example 38.1 that a 1.91-eV photon has a wavelength of 650 nm. Here the electron energy, and therefore the x-ray photon energy, is 10.0×10^{3} eV = 10.0 keV, about 5000 times greater than in Example 38.1, and the wavelength is about $\frac{1}{5000}$ as great as in Example 38.1. This makes sense, since wavelength and photon energy are inversely proportional.

Applications of X Rays

X rays have many practical applications in medicine and industry. Because x-ray photons are of such high energy, they can penetrate several centimeters of solid matter. Hence they can be used to visualize the interiors of materials that are opaque to ordinary light, such as broken bones or defects in structural steel. The object to be visualized is placed between an x-ray source and an electronic detector (like that used in a digital camera). The darker an area in the image recorded by such a detector, the greater the radiation exposure. Bones are much more effective x-ray absorbers than soft tissue, so bones appear as light areas. A crack or air bubble allows greater transmission and shows as a dark area.

38.9 This radiologist is operating a CT scanner (seen through the window) from a separate room to avoid repeated exposure to x rays.



A widely used and vastly improved x-ray technique is *computed tomography*; the corresponding instrument is called a *CT scanner.* The x-ray source produces a thin, fan-shaped beam that is detected on the opposite side of the subject by an array of several hundred detectors in a line. Each detector measures absorption along a thin line through the subject. The entire apparatus is rotated around the subject in the plane of the beam, and the changing photon-counting rates of the detectors are recorded digitally. A computer processes this information and reconstructs a picture of absorption over an entire cross section of the subject (see **Fig. 38.9**). Differences in absorption as small as 1% or less can be detected with CT scans, and tumors and other anomalies that are much too small to be seen with older x-ray techniques can be detected.

X rays cause damage to living tissues. As x-ray photons are absorbed in tissues, their energy breaks molecular bonds and creates highly reactive free radicals (such as neutral H and OH), which in turn can disturb the molecular structure of proteins and especially genetic material. Young and rapidly growing cells are particularly susceptible, which is why x rays are useful for selective destruction of cancer cells. Conversely, however, a cell may be damaged by radiation but survive, continue dividing, and produce generations of defective cells; thus x rays can *cause* cancer.

Even when the organism itself shows no apparent damage, excessive exposure to x rays can cause changes in the organism's reproductive system that will affect its offspring. A careful assessment of the balance between risks and benefits of radiation exposure is essential in each individual case.