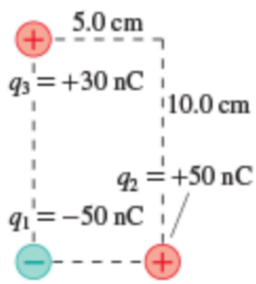
Three charged particles with $q_1 = -50\ \text{nC}$, $q_2 = +50\ \text{nC}$, and $q_3 = +30\ \text{nC}$ are placed on the corners of the 5.0 cm × 10.0 cm rectangle shown in Figure 22.18. What is the net force on charge q_3 due to the other two charges? Give your answer both in component form and as a magnitude and direction.

Figure 22.18

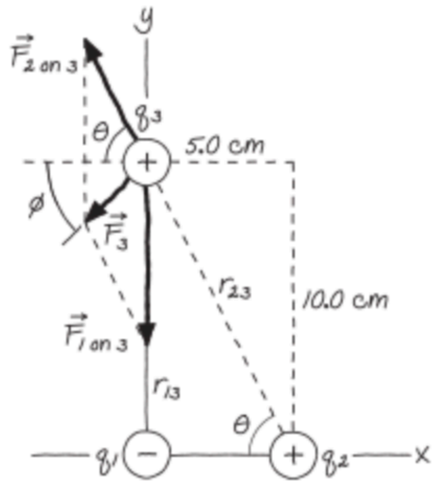


The three charges.

MODEL Model the charged particles as point charges.

VISUALIZE The pictorial representation of Figure 22.19 establishes a coordinate system. q_1 and q_3 are opposite charges, so force vector $\vec{F}_{1\text{ on }3}$ is an attractive force toward q_1. q_2 and q_3 are like charges, so force vector $\vec{F}_{2\text{ on }3}$ is a repulsive force away from q_2. q_1 and q_2 have equal magnitudes, but $\vec{F}_{2\text{ on }3}$ has been drawn shorter than $\vec{F}_{1\text{ on }3}$ because q_2 is farther from q_3. Vector addition has been used to draw the net force vector $\vec{F}_3$ and to define its angle ϕ.

Figure 22.19



A pictorial representation of the charges and forces.

SOLVE The question asks for a *force*, so our answer will be the *vector* sum $\vec{F}_3 = \vec{F}_{1\text{ on }3} + \vec{F}_{2\text{ on }3}$. We need to write $\vec{F}_{1\text{ on }3}$ and $\vec{F}_{2\text{ on }3}$ in component form. The magnitude of force $\vec{F}_{1\text{ on }3}$ can be found using Coulomb's law:

$$
\begin{aligned}
F_{1\text{ on }3} &= \frac{K|q_1||q_3|}{{r_{13}}^2} \\
&= \frac{(9.0 \times 10^9\ \mathrm{N\,m^2/C^2})(50 \times 10^{-9}\ \mathrm{C})(30 \times 10^{-9}\ \mathrm{C})}{(0.100\ \mathrm{m})^2} \\
&= 1.35 \times 10^{-3}\ \mathrm{N}
\end{aligned}
$$

where we used $r_{13} = 10.0$ cm.

The pictorial representation shows that $\vec{F}_{1\text{ on }3}$ points downward, in the negative y-direction, so

$$\vec{F}_{1\,\text{on}\,3} = -1.35 \times 10^{-3}\hat{j}\,\text{N}$$

To calculate $\vec{F}_{2\,\text{on}\,3}$ we first need the distance r_{23} between the charges:

$$r_{23} = \sqrt{(5.0\,\text{cm})^2 + (10.0\,\text{cm})^2} = 11.2\,\text{cm}$$

The magnitude of $\vec{F}_{2\,\text{on}\,3}$ is thus

$$\begin{aligned} F_{2\,\text{on}\,3} &= \frac{K\,|q_2|\,|q_3|}{{r_{23}}^2} \\ &= \frac{(9.0 \times 10^9\,\text{N}\,\text{m}^2/\text{C}^2)(50 \times 10^{-9}\,\text{C})(30 \times 10^{-9}\,\text{C})}{(0.112\,\text{m})^2} \\ &= 1.08 \times 10^{-3}\,\text{N} \end{aligned}$$

This is only a magnitude. The *vector* $\vec{F}_{2\,\text{on}\,3}$ is

$$\vec{F}_{2\,\text{on}\,3} = -F_{2\,\text{on}\,3}\cos\theta\hat{\imath} + F_{2\,\text{on}\,3}\sin\theta\hat{\jmath}$$

where angle θ is defined in the figure and the signs (negative x-component, positive y-component) were determined from the pictorial representation. From the geometry of the rectangle,

$$\theta = \tan^{-1}\left(\frac{10.0\,\text{cm}}{5.0\,\text{cm}}\right) = \tan^{-1}(2.0) = 63.4°$$

Thus $\vec{F}_{2\,\text{on}\,3} = (-4.83\hat{\imath} + 9.66\hat{\jmath}) \times 10^{-4}\,\text{N}$. Now we can add $\vec{F}_{1\,\text{on}\,3}$ and $\vec{F}_{2\,\text{on}\,3}$ to find

Thus $\vec{F}_{2\,\text{on}\,3} = (-4.83\hat{i} + 9.66\hat{j}) \times 10^{-4}$ N. Now we can add $\vec{F}_{1\,\text{on}\,3}$ and $\vec{F}_{2\,\text{on}\,3}$ to find

$$\vec{F}_3 = \vec{F}_{1\,\text{on}\,3} + \vec{F}_{2\,\text{on}\,3} = (-4.83\hat{\imath} + 3.84\hat{j}) \times 10^{-4}\ \text{N}$$