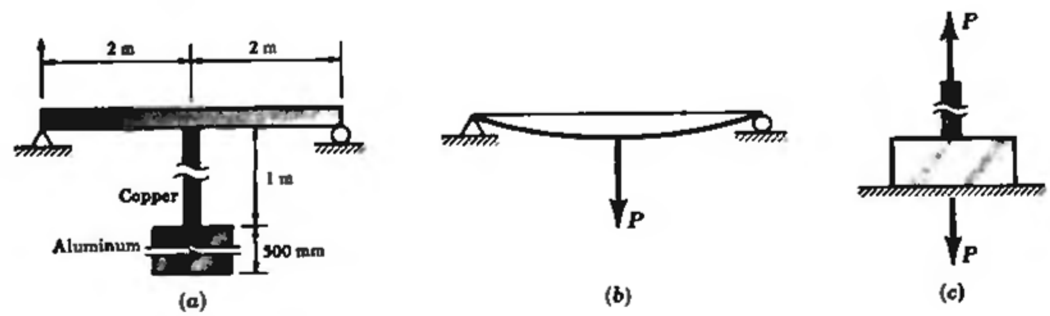

Fig. 11-9

A free-body diagram of the horizontal beam appears as in Fig. 11-9(*b*). Here, P denotes the force exerted upon the beam by the copper rod. Since this force is initially unknown, there are three forces acting upon the beam, but only two equations of equilibrium for a parallel force system; hence the problem is statically indeterminate. It will thus be necessary to consider the deformations of the system.

A free-body diagram of the two vertical rods appears as in Fig. 11-9(*c*). The simplest procedure is temporarily to cut the connection between the beam and the copper rod, and then allow the vertical rods to contract freely because of the decrease in temperature. If the horizontal beam offers no restraint, the copper rod will contract an amount

$$\Delta_{cu} = (20 \times 10^{-6})(10^3)(40) = 0.8 \text{ mm}$$

and the aluminum rod will contract by an amount

$$\Delta_{al} = (25 \times 10^{-6})(500)(40) = 0.5 \text{ mm}$$

However, the beam exerts a tensile force P upon the copper rod and the same force acts in the aluminum rod as shown in Fig. 11-9(*c*). These axial forces elongate the vertical rods and this elongation (see Problem 1.1) is

$$\frac{P(10^3)(10^6)}{500(100 \times 10^9)} + \frac{P(500)(10^6)}{10^3(70 \times 10^6)}$$

The downward force P exerted by the copper rod upon the horizontal beam causes a vertical deflection of the beam. In Problem 9.12 this central deflection was found to be $\Delta = PL^3/48EI$.

Actually, of course, the connection between the copper rod and the horizontal beam is not cut in the true problem and we realize that the resultant shortening of the vertical rods is exactly equal to the downward vertical deflection of the midpoint of the beam. This change of length of the vertical rods is caused partially by the decrease in temperature and partially by the axial force acting in the rods. For the shortening of the rods to be equal to the deflection of the beam we must have

$$(0.8 + 0.5) - \left[\frac{P(10^3)(10^6)}{500(100 \times 10^9)} + \frac{P(500)(10^6)}{10^3(70 \times 10^9)}\right] = \frac{P(4 \times 10^3)^3(10^6)}{48(10 \times 10^9)(400 \times 10^6)}$$

Solving, $P = 3.61$ kN; then,

$$\sigma_{cu} = 3.61 \times 10^3/500 = 7.22 \text{ MPa} \qquad \text{and} \qquad \sigma_{al} = 3.61 \times 10^3/1000 = 3.61 \text{ MPa}$$

11.6. The beam of flexural rigidity EI shown in Fig. 11-10 is clamped at both ends and subjected to a uniformly distributed load extending along the region BC of length $0.6L$. Determine all reactions.

At end A as well as C the supporting walls exert bending moments M_A and M_C plus shearing forces R_A and R_C as shown. For such a plane, parallel force system there are two equations of static equilibrium

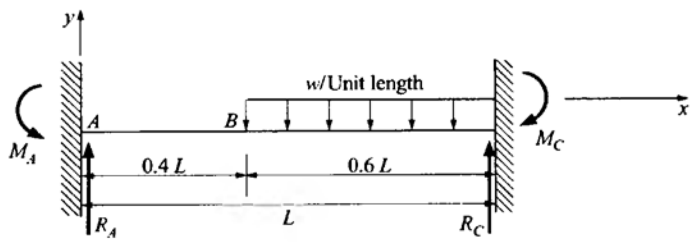

Fig. 11-10

and we must supplement these equations with additional relations stemming from beam deformations. The bending moment along the length ABC is conveniently written in terms of singularity functions:

$$EI\frac{d^2y}{dx^2} = -M_A\langle x\rangle^0 + R_A\langle x\rangle - \frac{w\langle x-0.4L\rangle^2}{2} \tag{1}$$

Integrating,

$$EI\frac{dy}{dx} = -M_A\langle x\rangle^1 + R_A\frac{\langle x\rangle^2}{2} - \frac{w}{2}\frac{\langle x-0.4L\rangle^3}{3} + C_1 \tag{2}$$

where C_1 is a constant of integration. As the first boundary condition, we have: when $x = 0$, the slope $dy/dx = 0$. Substituting in Eq. (2), we have

$$0 = -0 + 0 - 0 + C_1 \qquad \text{for} \qquad C_1 = 0$$

As the second boundary condition, when $x = L$, $dy/dx = 0$. Substituting in Eq. (2), we find

$$0 = -M_A L + \frac{R_A L^3}{2} - \frac{w}{6}(0.6L)^3 \tag{3}$$

Next, integrating Eq. (2), we find

$$EIy = -M_A\frac{\langle x\rangle^2}{2} + \frac{R_A}{2}\frac{\langle x\rangle^3}{3} - \frac{w}{6}\frac{\langle x-0.4L\rangle^4}{4} + C_2 \tag{4}$$

The third boundary condition is: when $x = 0$, $y = 0$, so from Eq. (4) we have $C_2 = 0$. The fourth boundary condition is: when $x = L$, $y = 0$, so from Eq. (4) we have

$$0 = -\frac{M_A L^2}{2} + \frac{R_A L^3}{6} - \frac{w}{24}(0.6L)^4 \tag{5}$$

The expressions for M_A given in Eqs. (3) and (5) may now be equated to obtain a single equation containing R_A as an unknown. Solving this equation, we find

$$R_A = wL\left\{(0.6)^3 - \frac{(0.6)^4}{2}\right\}$$
$$= 0.1512wL$$

Substituting this value in Eq. (3), we find $M_A = 0.0396wL^2$.

From statics we have

$$\Sigma F_y = -(0.6L)w + 0.1512wL + R_C = 0 \qquad \therefore R_C = 0.4488wL$$

and

$$+\circlearrowright\ \Sigma M_A = -0.0396wL^2 - M_C + (0.4488wL)(L) - [w(0.6L)](0.7L) = 0$$
$$\therefore M_C = 0.0684wL^2$$

11.7. The beam in Fig. 11-11 of flexural rigidity EI is clamped at A, supported between knife edges at B, and loaded by a vertical force P at the unsupported tip C. Determine the deflection at C.