Bonus

Monday, February 23, 2009
2:13 PM

Exercise 6.12 *(6 points.)* Let R be a ring, and suppose that $a^3 = a$ for each $a \in R$. Show that R is commutative.

Show that $ab = ba$

$ab \in R$ so $(ab)^3 = ab$

$(ab)(ab)(ab) = ab$

$a(ba)(ba)b = ab$

$a(ba)^2 b = ab \longrightarrow$ can only be if $(ba)^2 = e$.

$(ba)^3 = ba = (ba)(ba)(ba)$

$\Rightarrow b(ab)(ab)a = ba = b(ab)^2 a = ba$

this can only be if $(ab)^2 = e$

where e is an identity element with respect to multiplication.

$(ba)^2 (ab)^2 =$

$(ba)(ba)(ab)(ab) =$

$bab(aa)bab = bab\, a^2 bab = bab^2ab = ba^2b = b^2 = e$

So since each element multiplied by itself is equal to our multiplicative identity.

$\Rightarrow (ba)^2 = (ab)^2$

$\Rightarrow (ba)^2 = b^2a^2 = baba = bbaa = aba = baa = ab = ba.$

So we have proven that $ab = ba$.

Exercise 6.12 *(6 points.)* Let R be a ring, and suppose that $a^3 = a$ for each $a \in R$. Show that R is commutative.

let $a, b \in R$. then $(a+b) \in R$.

$(a+b)^3 = a+b$.

$\Rightarrow (a^2+ab+ba+b^2)(a+b) = \overset{a}{\cancel{a^3}} + aba + b\cancel{a^2} + \cancel{b^2}a + \cancel{a^2}b + a\cancel{b^2} + bab + \overset{b}{\cancel{b^3}}$

$= ba(a+b) + ab(a+b) + aba + bab + a + b$

$(ba+ab)(a+b) + (a+b)$

$[(ba+ab)+1](a+b) + aba + bab = a+b$

$aba + bab = e_+$

$ba + ab + 1 = e_\times$