Ready, SET, Go!

A Research-Based Approach to Problem Solving

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If there is one thing that physics educators are likely to agree on, it is the notion that learning problem-solving skills is one of the primary goals of physics education. In this article, I propose a set of problem-solving approaches—and separately, a set of related teaching strategies—based on the research project that I have recently completed.[1]

Before I go any further, I would like to stress the importance of distinguishing between *problems* and *exercises*. What definition of "problem" is useful when it comes to teaching problem-solving skills? Such a definition is a collective value judgment of many respected physics educators who have written on this subject. For instance, the article entitled "Exercises Are Not Problems" appeared in this journal more than 30 years ago.[2] A problem, according to the author of that paper, is something that "puzzles and worries. The solution cannot follow any logical procedure, for if it does, the problem is not a problem but an exercise in following that procedure. Hence the obstacle to be overcome in problem solving must be a logical gap" (p. 236). Lawson & Wollman[3] suggest that a problem-solving process must produce "contradictions … [that] produce the state of disequilibrium … patterns of reasoning are found wanting and must somehow be changed" (p. 470). The authors then suggest that problems must be such that "the student can partially but not completely understand them in terms of old ideas…; and sufficient time must be allowed … to grapple with the new situation, possibly with appropriate 'hints'" (p. 471). Fuller[4] writes: "To develop reasoning, people need to be puzzled by their own experiences, not by the … explanations given by a teacher …. Classroom exercises need to be designed to be slightly puzzling to the students given their present mental constructs. The students need to confront these puzzles in an environment where understanding them makes a difference … for their own self-esteem, self-confidence and mental equilibrium" (p. 47).

To sum up, a near-universal agreement that emerges from the literature is that a problem should be defined as an intellectually *challenging puzzle*, in which the path to the solution is *not initially obvious* to the solver. The solving process for such a task is not algorithmic in nature and cannot be easily codified. On the other hand, *an exercise* can be reasonably defined as a task that can be solved using a learned algorithm, a task where the broad solution path is fairly obvious to the student in the very beginning of the process.

The difference between exercises and problems is substantial; in fact, I argue that these tasks require substantially *different cognitive skills* to be solved.[1] Students need to be taught both, of course. However, my research shows that the lack of articulated distinction between problems and exercises in the problem-solving literature has led to a strange situation: almost all of the "problem-solving" advice found in journals, magazines[1] or online[5] is more relevant to *exercise* solving. Classroom instruction also appears to be heavily focused on exercises. Typically, students are taught topic-specific solving algorithms (first, label all the forces; second, draw the axes) rather than general solving strategies helpful in confronting the situations where "patterns of reasoning are found

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wanting." Most textbooks are also exercise oriented. For instance, virtually every one of them contains "problem-solving boxes" of some kind for each major topic. Since they do not teach *general* solving strategies, they would be more aptly named "exercise-solving boxes"—precisely because a different one appears in *each* chapter!

As a result of such lopsided research, instruction practices, and textbook content, students are often unprepared for tasks that require even a slight change of the algorithm. They don't like such tasks and sometimes express their disdain for them quite explicitly: "Hey, we have *never* done this stuff before—it is just not *fair* to give it on the test!" These cries signal that the "logical gap" has opened up, that "patterns of reasoning are wanting"—in other words, the student has just identified the task at hand as a *problem*—and rebelled. Justifiably so, I may add: one needs to be *taught* what to do when the task is "unfair."

This article aims to give the students and their teachers some additional ammunition for dealing with such "unfair" tasks—it is strictly about *problem* solving. In my research, I analyzed the mental mechanisms of problem solving using more than a hundred high school students enrolled in Advanced Placement (AP) Physics C, a calculus-based course. The participants solved several tasks that were administered over the Internet using an interactive platform, CyberTutor.[6] The tasks included a multiple-choice test and five open-response problems. In addition, eight participants solved four problems each in one-on-one sessions with me. Most of the problems, which were more challenging than typical AP-level tasks, were selected from one of my books.[7] The study helped identify the mental mechanisms involved in solving challenging tasks and the related solving strategies used by successful problem solvers. Based on the results of the study, I also propose some ideas for teaching. Being acutely aware that no idea in physics education research can be successful unless it is backed by a reasonably catchy acronym, I have decided to express the three components of successful teaching process as **SET**: **S**trategies, **E**xample, **T**asks.

Strategies

When students come to a physics class, they have had much experience with what they *think* was problem solving; realistically, it means that they have a suitcase full of misconceptions regarding the solving process itself. These misconceptions must be addressed, and ideally, replaced by effective problem-solving approaches. One of the important results of the study was that the more-successful problem solvers tend to use strategies that are *very similar* to the ones used by the less-successful solvers. However, stronger problem solvers use these strategies more effectively and consistently. My study has identified a number of such solving strategies. To better serve the purpose of this article, I present them as an "instruction sheet" for the students.

- Before you approach a problem, you must assume that it will be challenging. You may have to spend quite a bit of time solving it; your initial approach may well fail. This is normal, and you should not feel discouraged.
- Before writing anything, read the problem slowly and carefully. Ponder the problem, visualize the described situation, think about the knowledge that you may find useful in solving it. Identify the *general* physics principles that may be relevant. For example: "Energy seems to be conserved here."
- If the situation looks totally unfamiliar, try to look for *at least some* familiar features within it; your experience *should* be sufficient for finding them.
- Ideally, you should have a path to the answer broadly mapped out before you proceed to look for specific equations. Even if this path eventually leads nowhere, it helps to have it. If such "advance mapping" seems too hard to do, at least use short-term planning. Whenever you consider another step, ask yourself: What is next? Why am I trying to find this? Will this help?
- If the problem is quantitative in nature, make sure that you understand the way in which the desired answer can be expressed mathematically.
- Once the unknown is expressed mathematically, create an equation, or depending on the need, a system of equations that can be solved to find the unknown. This is the key part of the process: *expect to have difficulties here.* You will have to use the most general physics principles as well as the very specific inner features of the problem situation to elicit the necessary equations.

- Any interesting, nontrivial "special relationship" that you can identify within the problem situation may be turned into an equation and used in your solution. Example: "The blocks are connected by a string; therefore, their accelerations must be the same." *Finding "special relationships"* is the most challenging part of the solution process: be creative and persistent. You may have to rephrase the question, consider special cases, solve a sub-problem, etc. Sometimes, a *backwards approach* works best. What relationship would yield the answer directly? Can such a relationship (or equation) be found from the given information?
- On the other hand, even a generally correct equation may not be applicable to the specific problem situation or it may not be relevant to the solution. *Question the relevance of any equation* that you enter into your system of equations.
- If you cannot assemble the necessary equations, *consider an alternative approach:* Search for the inner features of the situation that may be more helpful than the ones you considered before; try to apply the concepts that are applicable but that you have not yet considered. Of course, it helps to know your basics.
- If you feel that you have enough equations to start solving, proceed with caution. Pause to inspect your intermediate results—do they make sense? If not, is it a technical error or a wrong assumption made when the equation was first constructed?
- Do not be afraid to introduce new unknowns. Either you will be able to solve for them or they will not be a part of the final answer.
- If you discover an error or hit a dead end, please remember: this is normal. Retrace your solution to the initial equations. Are they *relevant*? Are they based on the *correct* assumptions? Are there *enough* of them to produce an answer? Can some of them be replaced? To replace them, you may need to discover something else about the problem situation.
- *Go back* to the problem statement and search for the "special relationships" that you may have missed. This may take some time. If the problem situation is too complex to grasp, think about ways to simplify it. Solve a simpler sub-problem, then come back to the original one.
- If you finally assemble a solvable system of equations and solve it, examine the answer. If the answer is numeric, is the value obtained realistic? If algebraic, is it dimensionally correct? Does it hold true for extreme cases? Are the initial assumptions reasonable?
- Reflect. What does the answer mean? What is it useful for? Could this problem be solved in a different way?
- You just solved a tough problem. Listen to your feelings. You went through disappointments, dead ends, frustration—but you solved it. Wasn't it fun?

Of course, this list is not meant to be perfect or exhaustive. Every instructor is welcome to "pick and choose" to fit his or her personal style. However, a few general principles need to be followed. First, the strategies that you do find useful should be taught *explicitly* (as many physics educators have suggested before).[8] Second, it is important that these strategies remain fairly general and not related to any one physics topic—otherwise, they risk becoming *exercise-solving* instructions. Third, the necessity of "going back" and the acceptance of *temporary* failures need to be strongly emphasized. Just convincing the students to be persistent—in an intelligent and structured way—goes a long way toward improving their confidence and problem-solving skills. Fourth, you must show your students how you would use these strategies when challenged, which brings us to the second component of the SET approach.

Example

The students must see the solving strategies in action in order to be "sold" on their effectiveness and to begin learning them. Therefore, it is imperative for the instructor to *lead by example.* To my knowledge, most instructors fail to do so. When they demonstrate "problem-solving examples" in class, they always seem to know what the next step is: their solutions are always sequential, neat, and smooth. No mistakes are ever made, no hesitations shown, and the final answer is always right, of course.

My strong opinion is that such a method of teaching problem-solving skills is, in fact, counterproductive. The "flawless" solutions leave the students confused and unprepared for confronting a task that is more than one step away from the example they just saw. A common reaction to such solutions, either demonstrated by the teacher or shown in the textbook, is: "I understand these steps now that I see them; however, I still have no idea how they came up with those steps." Well, show them. Pick a new, unfamiliar, *challenging* (for you!) problem to solve in front of the students. Do not be afraid to struggle. Do not be afraid to embarrass yourself. Do make mistakes. Go back. Check yourself. Ask the students for help. Try again. In general, show the students how *you* use the strategies that you just recommended when *you* need to solve a difficult task. If you are too good a problem solver to find a problem that presents a real challenge for you, *pretend* to be challenged: a little acting has always been a part of the teaching craft. Show the students how one can recover from a dead end or an error. Since being able to find an alternative approach may be a key to solving a problem, spend some time and show more than one solution to a problem. Many students have no idea that a problem may be approached in more than one way.

More importantly, make it clear that you *enjoy* the process (you *do* like solving problems, right?). Many of your students have never experienced the joy that comes from cracking a problem that they thought they couldn't do. Helping them discover that joy may be the best thing you can do for them. They will seek it again and again by attacking new challenges tenaciously and intelligently (OK, maybe I am being a tad optimistic here). To sum up, your own behavior and attitude should emulate the one that you would like to see in your own students.

Of course, merely *demonstrating* a problem-solving process does little unless the students themselves get to solve problems. Larkin notes that "[v]irtually unanimously, individuals report acquiring skills through practice"[9] (pp. 534–535). You should plan classroom activities that facilitate problem-solving approaches that you want to teach—and put your money where your mouth is. Incentives are important. You can preach the value of the "process" all you want, but the students will ignore your words unless they are backed by the grading policies that show what you *really* care about.[10] Show the students that using the effective strategies *is* good for their grade. Do not just grade their solutions for "correctness." Commend your students—and give *formal* credit—for risk-taking, for spending much time on one problem, for coming up with multiple solutions, for noticing a hidden clue, etc. Be supportive, optimistic, and sensitive. Solving a problem requires, by definition, bold leaps into the unknown. Some of these leaps lead to crash landings, and you need to be there to help the students glue their fragile egos together. Inspire, excite, and remain patient. To reduce stress and to enrich the experience, it helps to have the students work in small groups.[11] That way, they can observe and support each other as they work on finding the small pieces of an exciting physics puzzle that you just gave them.

Tasks

Speaking of exciting puzzles, one of the biggest mistakes that an instructor can make is to introduce a hefty list of problem-solving strategies and then offer students a series of trivial, exercise-like tasks to practice. This is tempting (let us go easy first), but this just doesn't work. Many educators who attempted explicit teaching of problem-solving strategies later reported, with some frustration, that their students flatly rejected the "expert" strategies and reverted to the "plug-and-chug" approach. Why would the students do that?

The answer is simple: because "plug-and-chug" *worked* for the tasks that they were given! Problem-solving strategies are neither necessary nor particularly useful in solving trivial exercises. To learn to solve problems, the students need to be given both the right tools (problem-solving strategies) and the right material (challenging tasks). Where does one get a supply of such tasks? First of all, the textbooks should not be dismissed as their source. Despite the lack of truly interesting tasks, textbooks contain plenty of tasks that, given the state of knowledge of the students at a certain point in the course, could be challenging enough to be viewed as *problems.*

Many other sources exist as well. Large numbers of interesting problems have been published in various books and magazine articles.[7, 12-20] There are also problems from physics competitions of various levels, widely available on the Internet, and of course, the monthly column of *Physics Challenges* appearing in this journal.[21] The selection of available interesting problems is wide, if one really wants to find them. It is up to the instructor to mix problems and exercises judiciously, and carefully judge the probable width of the "logical gap" encountered by the students, but one must leave this gap for the students to jump across. They'll enjoy it—eventually.

To conclude: it is my sincere hope that my fellow teachers would find some of these ideas useful. I must confess that my project was limited to studying the *cognition* of problem solving; I did not explore the effects of *teaching* problem-solving skills. While the SET teaching approach is tightly based on my research results, its empirical validation (or lack thereof) is a matter of the future. I would highly appreciate any feedback.

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