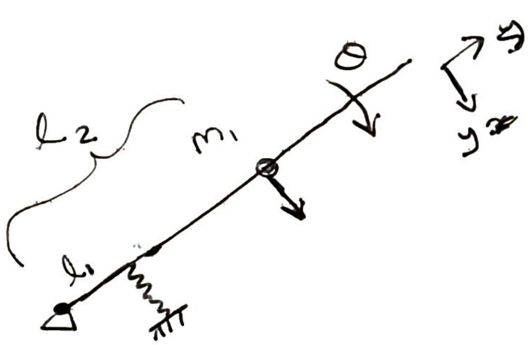

m_1 = mass of system

tangential velocity = $\ell_2\dot{\theta}$

$k = \frac{1}{2}mv_t^2 = \frac{1}{2}m(\ell_2\dot{\theta})^2 = \frac{1}{2}m\ell_2^2\dot{\theta}^2$ (translational)

$k = \frac{1}{2}I\dot{\theta}^2$ (rotational)

$T = \frac{1}{2}m\ell_2^2\dot{\theta}^2 + \frac{1}{2}I\dot{\theta}^2 = \frac{1}{2}\dot{\theta}^2(m\ell_2^2 + I)$ / $D = \frac{1}{2}c\dot{\theta}^2$

$U = \frac{1}{2}kx^2$ where $x = \theta\ell_1$

$= \frac{1}{2}k\theta^2\ell_1^2$

$\mathcal{L} = T - U$; $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} - \frac{d}{dt}\left(\frac{\partial U}{\partial \dot{q}}\right) + \frac{\partial U}{\partial q} + \frac{\partial D}{\partial \dot{q}} = 0$

$\frac{\partial T}{\partial \dot{\theta}} = \dot{\theta}(m\ell_2^2 + I)$ $\quad$ $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = \ddot{\theta}(m\ell_2^2 + I)$

$\frac{\partial T}{\partial \theta} = 0$ $\quad$ $\frac{\partial D}{\partial \dot{\theta}} = c\dot{\theta}$

$\frac{\partial U}{\partial \dot{\theta}} = 0$ $\quad$ $\frac{d}{dt}\left(\frac{\partial U}{\partial \dot{\theta}}\right) = 0$ $\quad$ $\frac{\partial U}{\partial \theta} = k\theta\ell_1^2$

$\mathcal{L} = \ddot{\theta}(m\ell_2^2 + I) + k\theta\ell_1^2 + c\dot{\theta} = 0$

$$\ddot{\theta} = \frac{-k\theta\ell_1^2 - c\dot{\theta}}{m\ell_2^2 + I}$$