Hello and thank you for your reply! However I have below problems.

In the section 1.9 of the above mentioned book the author talks about exact equations and he writes them in the below form.

$\mathrm{M(t,y)} + \mathrm{N(t,Y)}\frac{dy}{dt} = 0$ (I)

Then he points out if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$ the above equations could be considered as an exact equation and it has a solution however, if $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial t}$ then the equation (I) could be turned into an exact equation and then be solved only if the below exists:

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial t}}{N} = \ a\ function\ of "t"\ only$$

or

$$\frac{\frac{\partial N}{\partial t} - \frac{\partial M}{\partial y}}{M} = a\ function\ of\ "y"\ only$$

And there exists an "integrating factor" of the form

$exp\left(\int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial t}}{N}.dt\right)$ or $exp\left(\int \frac{\frac{\partial N}{\partial t} - \frac{\partial M}{\partial y}}{M}.dy\right)$

Now my problems:

Consider Bernoulli equation of below:

$$\frac{dy}{dt} + a(t).y = b(t).y^n$$

First why $\mu(t) = \exp(\int a(t).dt)$ is multiplied to the both sides of the equation? (Note: I understand $\mu(t) = \exp(\int a(t).dt)$ is integration factor for first order linear differential equations but why it is multiplied to both sides of Bernoulli equation (is Bernoulli equation a first order linear differential equation? Because I don't think so!)

Second, Consider below equation:

$$\frac{d}{dt}(\mu(t).y) = b(t).\mu(t).y^n$$

If we multiply both sides of the equation by $(1-n)y^{-n}$

We would have:

$$-(1-n)b(t)\mu(t) + (1-n)y^{-n}.\frac{d}{dt}(\mu(t).y) = 0$$

Now if we assume

$$\mathrm{M(t,y)} = -(1-n)b(t)\mu(t) \; \& \; \mathrm{N(t,Y)} = (1-n)y^{-n}.\frac{d}{dt}(\mu(t).y)$$

Then how could we calculate integrating factor? (As the question requires)

I think we should consider μ(t) (as it is appeared in the question) what do u say?