B.Sc (Hons) Physics Semester IV
$\mathrm{II}^{nd}$ Sessional Examination, 2021

**Quantum Mechanics**

Paper Code : PHB 42C

*Time : 1 hr* *Maximum Marks : 20*

Answer any 4 questions. All questions carry equal marks (5 × 4)

1. (i) If $\hat{A}$ and $\hat{B}$ commute and if $|\psi_1>$ and $|\psi_2>$ are two eigen vectors of $\hat{A}$ with two eigen values ($\hat{A}$ is hermitian), show that (a) $,\psi_1|B|\psi_2>$ is zero and (b) $\hat{B}|\psi_1>$ is also an eigen vector of $\hat{A}$ and find its eigenvalue.

   (ii) Specify the nature of expression (operator, bra vector or ket vector) and then find its Hermitian conjugate
   (a) $<\phi|\hat{A}|\psi><\psi|$,
   (b) $\hat{A}|\psi><\phi|$,
   (c) $<\phi|\hat{A}|\psi>|\psi><\phi|\hat{A}$,
   (d) $<\psi|\hat{A}|\phi>|\phi>+i\hat{A}|\psi>$

2. (i)Prove

$$\frac{d}{dt}<\hat{A}\hat{B}>=<\frac{\partial\hat{A}}{\partial t}\hat{B}>+<\hat{A}\frac{\partial\hat{B}}{\partial t}>+\frac{1}{i\hbar}<[\hat{A},\hat{H}]\hat{B}>+\frac{1}{i\hbar}<\hat{A}[\hat{H},\hat{B}]>$$

   (ii) Show that $[\hat{x}^2,\hat{p}^2]=2i\hbar(i\hbar+2\hat{p}\hat{x})$

3. Prove

$$\frac{m\omega^2}{2}<n|x^2|n>=\frac{1}{2m}<n|p^2|n>=\frac{1}{2}<n|H|n>$$

   where $|n>$ is the eigen state of a harmonic oscillator.

4. Consider a system whose wave function at time $t=0$ is given by

$$\psi(x,0)=\frac{5}{\sqrt{50}}\phi_0(x)+\frac{4}{\sqrt{50}}\phi_1(x)+\frac{3}{\sqrt{50}}\phi_2(x)$$

   where $\phi_n(x)$ is the wave function of the $n^{th}$ excited state of a harmonic oscillator with energy $E_n=(n+\frac{1}{2})\hbar\omega$
   (a) Find the average energy of this systemin this state.
   (b) Find the state $\psi(x,t)$ at a later time $t$.

5. Consider a system in a state

$$\psi(\theta,\phi) = \frac{1}{2}Y_{00}(\theta,\phi) + \sqrt{\frac{1}{3}}Y_{11}(\theta,\phi) + \frac{1}{2}Y_{1,-1}(\theta,\phi) + \sqrt{\frac{1}{6}}Y_{2,2}(\theta,\phi)$$

(i) Is $\psi(\theta,\phi)$ normalised?
(ii) Is $\psi(\theta,\phi)$ an eigenstate of $L^2$ or $L_z$?

6. Calculate $J_- J_+ |j, m>$ and $J_+ J_- |jm>$ and then infer the commutator $[J_+ J_-, J_- J_+] |jm>$

7. Find the matrices representing the operators $\hat{J}^2$,$\hat{J}_z$ and $\hat{J}_+$ for the case $j = 3/2$.