# QM thread: Physical significance of a.$\sigma$ in expectation -E(a.$\sigma$ b.$\sigma$)?

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## 1 Appendix

4.1. From ¶2.18, we now provide an analogous local-realistic calculation of the expectation $\langle AB\,|\,E\rangle$. We allow that the pairwise conservation of total angular momentum in (4) $[\lambda_i + \mu_i = 0]$ reduces to the pairwise conservation of intrinsic spin $[\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2 = 0]$ during each particle's continued interaction with a polarizing-field. Each pair now emerges under the correlating influence of $\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2 = 0$; our results unchanged.

4.2. Let the intrinsic spin of a pristine spin-half particle be $\boldsymbol{s} = \frac{\hbar}{2}\boldsymbol{\sigma}$. So $\boldsymbol{s}$ has magnitude $\frac{\hbar}{2}$ in direction $\boldsymbol{\sigma}$; ie, $\boldsymbol{\sigma}$ is a direction-vector with $\sigma \equiv |\boldsymbol{\sigma}| = 1$. In the context of a pristine particle $q(\boldsymbol{\sigma})$ interacting with a polarizer $\delta^{\pm}_{\hat{a}}$, we now introduce and define the spin-product $\hat{a} \circ \boldsymbol{\sigma}$ where $\circ$ denotes the scalar product of $\hat{a}$ with $\boldsymbol{\sigma}$ after $\boldsymbol{\sigma}$ rotates to be parallel or antiparallel with $\hat{a}$. As is well known: $\boldsymbol{s}$ is then spin-up or spin-down with respect to $\hat{a}$ after $q(\boldsymbol{\sigma})$ interacts with a polarizer-field oriented $\hat{a}$. The spin-product is consistent with our use of $\lambda$ in (X2), etc:

$$q(\lambda) \Rightarrow \delta^{\pm}_{\hat{a}} \rightarrow q(\hat{a}^{\pm}) \Rightarrow [\hat{a}\cdot *] \rightarrow [\hat{a}\cdot\hat{a}^{\pm}] = \pm 1; \text{etc. } \hat{a}\circ\boldsymbol{\sigma} \equiv \pm\hat{a}\cdot\hat{a} = \sigma_{\hat{a}} = \pm 1; \left\langle (\hat{a}\circ\boldsymbol{\sigma})^2 \right\rangle = 1; \langle \hat{a}\circ\boldsymbol{\sigma}\rangle = 0. \quad (1)$$

$$\sigma^2 = \sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1 \because \sigma_x, \sigma_y, \sigma_z \text{ are rotations-onto, not projections-onto, a Cartesian frame.} \quad (2)$$

$$\langle \sigma_x\sigma_y \rangle = \langle \sigma_y\sigma_z \rangle = \langle \sigma_z\sigma_x \rangle = 0 \because \sigma_x, \sigma_y, \sigma_z \text{ take positive and negative values equiprobably; etc.} \quad (3)$$

$$\therefore\ \langle AB\,|\,E\rangle \equiv \left\langle (\hat{a}\circ\boldsymbol{\sigma}_1)(\hat{b}\circ(-\boldsymbol{\sigma}_2)) \right\rangle = -\left\langle (\hat{a}\circ\boldsymbol{\sigma}_1)(\hat{b}\circ\boldsymbol{\sigma}_2) \right\rangle = -\left\langle (\hat{a}\circ\boldsymbol{\sigma})(\hat{b}\circ\boldsymbol{\sigma}) \right\rangle \text{ (for convenience)} \quad (4)$$

$$= -(\hat{a}_x\ \hat{a}_y\ \hat{a}_z) \left\langle \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix} (\sigma_x\ \sigma_y\ \sigma_z) \right\rangle \begin{pmatrix} \hat{b}_x \\ \hat{b}_y \\ \hat{b}_z \end{pmatrix} = -(\hat{a}_x\ \hat{a}_y\ \hat{a}_z) \left\langle \begin{pmatrix} \sigma_x^2 & \sigma_x\sigma_y & \sigma_x\sigma_z \\ \sigma_y\sigma_x & \sigma_y^2 & \sigma_y\sigma_z \\ \sigma_z\sigma_x & \sigma_z\sigma_y & \sigma_z^2 \end{pmatrix} \right\rangle \begin{pmatrix} \hat{b}_x \\ \hat{b}_y \\ \hat{b}_z \end{pmatrix} \quad (5)$$

$$= -(\hat{a}_x\ \hat{a}_y\ \hat{a}_z) \begin{pmatrix} \langle\sigma_x^2\rangle & \langle\sigma_x\sigma_y\rangle & \langle\sigma_x\sigma_z\rangle \\ \langle\sigma_y\sigma_x\rangle & \langle\sigma_y^2\rangle & \langle\sigma_y\sigma_z\rangle \\ \langle\sigma_z\sigma_x\rangle & \langle\sigma_z\sigma_y\rangle & \langle\sigma_z^2\rangle \end{pmatrix} \begin{pmatrix} \hat{b}_x \\ \hat{b}_y \\ \hat{b}_z \end{pmatrix} = -(\hat{a}_x\ \hat{a}_y\ \hat{a}_z) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{b}_x \\ \hat{b}_y \\ \hat{b}_z \end{pmatrix} = -\hat{a}\cdot\hat{b}.\ QED. \quad (6)$$

4.3. As foreshadowed in ¶2.18, the generality of Malus' Law under CLR follows. From the fundamental definition of $\langle AB\,|\,E\rangle$, then using (6) and its independence from Malus, we have (X19) anew:

$$P(B^+\,|\,EA^+) = \tfrac{1}{2}(1 + \langle AB\,|\,E\rangle) = \tfrac{1}{2}(1 - \hat{a}\cdot\hat{b}) = \sin^2 \tfrac{1}{2}(\hat{a}, \hat{b}).\ QED.\ \blacksquare \quad (7)$$

4.4. Of interest: Fröhner 1998:(75), allowing spin $\boldsymbol{\sigma}_1 = -\boldsymbol{\sigma}_2$ to be ordinary vectors in 3-space, obtains:

$$\left\langle (\hat{a}\cdot\boldsymbol{\sigma}_1)(\boldsymbol{\sigma}_2\cdot\hat{b}) \right\rangle = -\left\langle (\hat{a}\cdot\boldsymbol{\sigma}_1)(\boldsymbol{\sigma}_1\cdot\hat{b}) \right\rangle = -\frac{\langle\sigma_1^2\rangle}{3}\,\hat{a}\cdot\hat{b} = -\hat{a}\cdot\hat{b} \text{ allowing } \left\langle\sigma_1^2\right\rangle = 3 \text{ under QM.} \quad (8)$$

4.5. Via our rotations-onto, no QM-based adjustment is required in (4)-(6). In Fröhner 1998:(75), our (8), working with projections-onto: $\left\langle\sigma_1^2\right\rangle = \left\langle\sigma_x^2\right\rangle + \left\langle\sigma_y^2\right\rangle + \left\langle\sigma_z^2\right\rangle = 3$.

## 2 References


1. Fröhner, F. H. (1998). "Missing link between probability theory and quantum mechanics: the Riesz-Fejér theorem." Z. Naturforsch. 53a, 637-654.