In what follows we will often drop the indices and simply write $iS(x-y) = \{\psi(x), \bar{\psi}(y)\}$, but you should remember that $S(x-y)$ is a $4\times 4$ matrix. Inserting the expansion (5.25), we have

$$\begin{aligned} iS(x-y) &= \int \frac{d^3p\, d^3q}{(2\pi)^6} \frac{1}{\sqrt{4E_{\vec{p}}E_{\vec{q}}}} \Big[ \{b^s_{\vec{p}}, b^{r\,\dagger}_{\vec{q}}\} u^s(\vec{p})\bar{u}^r(\vec{q}) e^{-i(p\cdot x - q\cdot y)} \\ &\qquad\qquad\qquad + \{c^{s\,\dagger}_{\vec{p}}, c^r_{\vec{q}}\} v^s(\vec{p})\bar{v}^r(\vec{q}) e^{+i(p\cdot x - q\cdot y)} \Big] \\ &= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} \left[ u^s(\vec{p})\bar{u}^s(\vec{p}) e^{-ip\cdot(x-y)} + v^s(\vec{p})\bar{v}^s(\vec{p}) e^{+ip\cdot(x-y)} \right] \\ &= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} \left[ (\not{p} + m) e^{-ip\cdot(x-y)} + (\not{p} - m) e^{+ip\cdot(x-y)} \right] \end{aligned} \tag{5.27}$$

where to reach the final line we have used the outer product formulae (4.128) and (4.129). We can then write

$$iS(x-y) = (i\not{\partial}_x + m)(D(x-y) - D(y-x)) \tag{5.28}$$

in terms of the propagator for a real scalar field $D(x-y)$ which, recall, can be written as (2.90)

$$D(x-y) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}}\, e^{-ip\cdot(x-y)} \tag{5.29}$$

Some comments:

- For spacelike separated points $(x-y)^2 < 0$, we have already seen that $D(x-y) - D(y-x) = 0$. In the bosonic theory, we made a big deal of this since it ensured that

$$[\phi(x), \phi(y)] = 0 \qquad (x-y)^2 < 0 \tag{5.30}$$

outside the lightcone, which we trumpeted as proof that our theory was causal. However, for fermions we now have

$$\{\psi_\alpha(x), \psi_\beta(y)\} = 0 \qquad (x-y)^2 < 0 \tag{5.31}$$

outside the lightcone. What happened to our precious causality? The best that we can say is that all our observables are bilinear in fermions, for example the Hamiltonian (5.17). These still commute outside the lightcone. The theory remains causal as long as fermionic operators are not observable. If you think this is a little weak, remember that no one has ever seen a physical measuring apparatus come back to minus itself when you rotate by 360 degrees!