Q1:

(a) $\begin{cases} \cancel{E_k = \frac{mv^2}{2} \quad (1)} \\ F = \frac{mv^2}{r} = \frac{e^2}{4\varepsilon_0 \pi r^2} \quad (2) \\ mvr = n\hbar \quad (3) \end{cases}$ for (2): $\frac{mv^2}{r} = \frac{m^2v^2r^2}{mr^3} = \frac{n^2\hbar^2}{mr^3} = \frac{e^2}{4\varepsilon_0\pi r^2}$.

$\Rightarrow r = \frac{4\pi\varepsilon_0}{e^2} \times \frac{n^2\hbar^2}{m} = \frac{4\pi\hbar^2\varepsilon_0}{me^2} \cdot n^2$.

$\because F = m\omega^2 r = 4\pi^2 f^2 mr = \frac{e^2}{4\varepsilon_0\pi r^2}$.

$\Rightarrow f^2 = \frac{e^2}{4\varepsilon_0\pi r^2} \cdot \frac{1}{4\pi^2 mr} = \frac{e^2}{16\pi^3 m\varepsilon_0} \cdot \frac{1}{r^3} = \frac{m^2e^8}{(4^2\pi^3)(64\pi^3)\hbar^6\varepsilon_0^4 \cdot n^6} \Rightarrow f_1 = \frac{me^4}{32\pi^3\hbar^3\varepsilon_0^2} \cdot \frac{1}{n^3}$

$\approx 6.57968 \times 10^{15} \times \frac{1}{n^3}$

(b) : $f_2 = \frac{me^4}{64\pi^3\hbar^3\varepsilon_0^2}\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right) \approx 6.57968 \times 10^{15} \cdot \frac{1}{2}\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)$.

$(E = hf)$

(c) They agree perfect! let $n = 200$, $f_1 = 8.22 \times 10^8$ $f_2 = 8.28 \times 10^8$.

let $n = 5000$, $f_1 = 52637.5$, $f_2 = 52653.3$

~~Infact~~ In fact, $\lim\limits_{n\to\infty} \frac{f_1}{f_2} = \frac{\frac{1}{n^3}}{\frac{1}{2}\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)} = 1$ since $\deg f_1 = \deg f_2$ and highest power coefficients equals each other. Therefore it satisfy the Bohr's correspondence principle.