

The net charge enclosed by the Gaussian surface in Fig. 25-16b is $q - q'$, so Gauss' law now gives

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = \epsilon_0 EA = q - q', \quad (25-32)$$

or

$$E = \frac{q - q'}{\epsilon_0 A}. \quad (25-33)$$

The effect of the dielectric is to weaken the original field E_0 by a factor of κ ; so we may write

$$E = \frac{E_0}{\kappa} = \frac{q}{\kappa \epsilon_0 A}. \quad (25-34)$$

Comparison of Eqs. 25-33 and 25-34 shows that

$$q - q' = \frac{q}{\kappa}. \quad (25-35)$$

Equation 25-35 shows correctly that the magnitude q' of the induced surface charge is less than that of the free charge q and is zero if no dielectric is present (because then $\kappa = 1$ in Eq. 25-35).

By substituting for $q - q'$ from Eq. 25-35 in Eq. 25-32, we can write Gauss' law in the form

$$\epsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q \quad (\text{Gauss' law with dielectric}). \quad (25-36)$$

This equation, although derived for a parallel-plate capacitor, is true generally and is the most general form in which Gauss' law can be written. Note:

1. The flux integral now involves $\kappa \vec{E}$, not just \vec{E} . (The vector $\epsilon_0 \kappa \vec{E}$ is sometimes called the *electric displacement* \vec{D} , so that Eq. 25-36 can be written in the form $\oint \vec{D} \cdot d\vec{A} = q$.)
2. The charge q enclosed by the Gaussian surface is now taken to be the *free charge only*. The induced surface charge is deliberately ignored on the right side of Eq. 25-36, having been taken fully into account by introducing the dielectric constant κ on the left side.
3. Equation 25-36 differs from Eq. 23-7, our original statement of Gauss' law, only in that ϵ_0 in the latter equation has been replaced by $\kappa \epsilon_0$. We keep κ inside the integral of Eq. 25-36 to allow for cases in which κ is not constant over the entire Gaussian surface.



Sample Problem 25.06 Dielectric partially filling the gap in a capacitor

Figure 25-17 shows a parallel-plate capacitor of plate area A and plate separation d . A potential difference V_0 is applied between the plates by connecting a battery between them. The battery is then disconnected, and a dielectric slab of thickness b and dielectric constant κ is placed between the plates as shown. Assume $A = 115 \text{ cm}^2$, $d = 1.24 \text{ cm}$, $V_0 = 85.5 \text{ V}$, $b = 0.780 \text{ cm}$, and $\kappa = 2.61$.

(a) What is the capacitance C_0 before the dielectric slab is inserted?

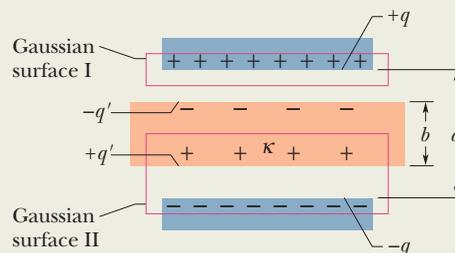


Figure 25-17 A parallel-plate capacitor containing a dielectric slab that only partially fills the space between the plates.

Calculation: From Eq. 25-9 we have

$$C_0 = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12} \text{ F/m})(115 \times 10^{-4} \text{ m}^2)}{1.24 \times 10^{-2} \text{ m}}$$

$$= 8.21 \times 10^{-12} \text{ F} = 8.21 \text{ pF.} \quad (\text{Answer})$$

(b) What free charge appears on the plates?

Calculation: From Eq. 25-1,

$$q = C_0 V_0 = (8.21 \times 10^{-12} \text{ F})(85.5 \text{ V})$$

$$= 7.02 \times 10^{-10} \text{ C} = 702 \text{ pC.} \quad (\text{Answer})$$

Because the battery was disconnected before the slab was inserted, the free charge is unchanged.

(c) What is the electric field E_0 in the gaps between the plates and the dielectric slab?

KEY IDEA

We need to apply Gauss' law, in the form of Eq. 25-36, to Gaussian surface I in Fig. 25-17.

Calculations: That surface passes through the gap, and so it encloses *only* the free charge on the upper capacitor plate. Electric field pierces only the bottom of the Gaussian surface. Because there the area vector $d\vec{A}$ and the field vector \vec{E}_0 are both directed downward, the dot product in Eq. 25-36 becomes

$$\vec{E}_0 \cdot d\vec{A} = E_0 dA \cos 0^\circ = E_0 dA.$$

Equation 25-36 then becomes

$$\epsilon_0 \kappa E_0 \oint dA = q.$$

The integration now simply gives the surface area A of the plate. Thus, we obtain

$$\epsilon_0 \kappa E_0 A = q,$$

or

$$E_0 = \frac{q}{\epsilon_0 \kappa A}.$$

We must put $\kappa = 1$ here because Gaussian surface I does not pass through the dielectric. Thus, we have

$$E_0 = \frac{q}{\epsilon_0 \kappa A} = \frac{7.02 \times 10^{-10} \text{ C}}{(8.85 \times 10^{-12} \text{ F/m})(1)(115 \times 10^{-4} \text{ m}^2)}$$

$$= 6900 \text{ V/m} = 6.90 \text{ kV/m.} \quad (\text{Answer})$$

Note that the value of E_0 does not change when the slab is introduced because the amount of charge enclosed by Gaussian surface I in Fig. 25-17 does not change.

(d) What is the electric field E_1 in the dielectric slab?

KEY IDEA

Now we apply Gauss' law in the form of Eq. 25-36 to Gaussian surface II in Fig. 25-17.

Calculations: Only the free charge $-q$ is in Eq. 25-36, so

$$\epsilon_0 \oint \kappa \vec{E}_1 \cdot d\vec{A} = -\epsilon_0 \kappa E_1 A = -q. \quad (25-37)$$

The first minus sign in this equation comes from the dot product $\vec{E}_1 \cdot d\vec{A}$ along the top of the Gaussian surface because now the field vector \vec{E}_1 is directed downward and the area vector $d\vec{A}$ (which, as always, points outward from the interior of a closed Gaussian surface) is directed upward. With 180° between the vectors, the dot product is negative. Now $\kappa = 2.61$. Thus, Eq. 25-37 gives us

$$E_1 = \frac{q}{\epsilon_0 \kappa A} = \frac{E_0}{\kappa} = \frac{6.90 \text{ kV/m}}{2.61}$$

$$= 2.64 \text{ kV/m.} \quad (\text{Answer})$$

(e) What is the potential difference V between the plates after the slab has been introduced?

KEY IDEA

We find V by integrating along a straight line directly from the bottom plate to the top plate.

Calculation: Within the dielectric, the path length is b and the electric field is E_1 . Within the two gaps above and below the dielectric, the total path length is $d - b$ and the electric field is E_0 . Equation 25-6 then yields

$$V = \int_-^+ E ds = E_0(d - b) + E_1 b$$

$$= (6900 \text{ V/m})(0.0124 \text{ m} - 0.00780 \text{ m})$$

$$+ (2640 \text{ V/m})(0.00780 \text{ m})$$

$$= 52.3 \text{ V.} \quad (\text{Answer})$$

This is less than the original potential difference of 85.5 V.

(f) What is the capacitance with the slab in place?

KEY IDEA

The capacitance C is related to q and V via Eq. 25-1.

Calculation: Taking q from (b) and V from (e), we have

$$C = \frac{q}{V} = \frac{7.02 \times 10^{-10} \text{ C}}{52.3 \text{ V}}$$

$$= 1.34 \times 10^{-11} \text{ F} = 13.4 \text{ pF.} \quad (\text{Answer})$$

This is greater than the original capacitance of 8.21 pF.

