Consider a continuous function $\psi : \mathbb{R}^n \to \mathbb{R}$,

$$\psi(x) = |x| + |f(x)|, \quad \psi \geq 0.$$

Observe that $\psi(x) \to \infty$ as $|x| \to \infty$.

Thus ψ attains its minimum in $\mathbb{R}^n$ say at x_0.

HYPOTHESIS 1. *There must be a constant $c > 0$ such that the set $K_c = \{\psi(x) \leq \psi(x_0) + c\}$ is homeomorphic to a closed ball.*

Observe that ψ is an invariant function: $\psi \circ f = \psi$. So that K_c is an invariant set: $f(K_c) \subset K_c$.

The Brouwer fixed point theorem finishes the proof.