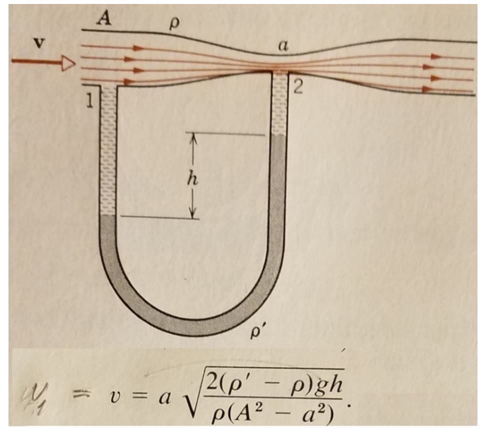
given $\frac{1}{2}\rho v^2 + p = constant \quad Av_1 = av_2$ $\quad \rho =$ density of water

$\rho ' =$ density of mercury



show $\quad v_1 = a\sqrt{\frac{2(\rho ' - \rho)gh}{\rho(A^2 - a^2)}}$

$$(1) \quad \frac{1}{2}\rho v_1^2 + p_1 = \frac{1}{2}\rho v_2^2 + p_2$$

Bernoulli eqn

$$(2) \quad \frac{1}{2}\rho v_1^2 + \rho gh = \frac{1}{2}\rho v_2^2 + \rho ' gh$$

sub p_x = ρgh, the static pressure term, in (1)

$$Av_1 = av_2 \quad \Rightarrow \quad v_2 = \frac{A}{a}v_1 \quad \Rightarrow \quad v_2^2 = \frac{A^2}{a^2}v_1^2$$

use continuity eqn to express v_2

$$(3) \quad \frac{1}{2}\rho v_1^2 + \rho gh = \frac{1}{2}\rho \frac{A^2}{a^2}v_1^2 + \rho ' gh$$

sub continuity result above in (2)

$$(4) \quad \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho \frac{A^2}{a^2}v_1^2 = \rho ' gh - \rho gh$$

collect v_1 terms and gh terms in (3)

$$(5) \quad \rho v_1^2 - \rho \frac{A^2}{a^2}v_1^2 = 2(\rho ' gh - \rho gh)$$

mult (4) by 2

$$(6) \quad \rho\left(1 - \frac{A^2}{a^2}\right)v_1^2 = 2(\rho ' - \rho)gh$$

factor out ρ and v_1^2 on lhs of (5) and factor out gh on rhs of (5)

$$(7) \quad v_1^2 = \frac{2(\rho ' - \rho)gh}{\rho(1 - A^2/a^2)}$$

divide (6) by $\rho(1 - A^2/a^2)$

$$(8) \quad v_1^2 = \frac{2(\rho ' - \rho)gh}{\rho(a^2 - A^2)1/a^2}$$

factor out $1/a^2$ in denominator of (7)

$$(9) \quad v_1^2 = a^2\frac{2(\rho ' - \rho)gh}{\rho(a^2 - A^2)}$$

simplify (8)

$$(10) \quad v_1 = a\sqrt{\frac{2(\rho ' - \rho)gh}{\rho(a^2 - A^2)}}$$

square root of (10)

My answer in (10) has $a^2 - A^2$ in the denominator, but it is supposed to be $A^2 - a^2$. I know I have shown more steps than are needed, but I have tried this ten times, and I am stumped. I even *reverse engineered* the problem and backed it out to $\frac{1}{2}\rho v_1^2 - \rho gh = \frac{1}{2}\rho v_2^2 - \rho ' gh$ which I can rearrange to $\frac{1}{2}\rho v_1^2 + \rho ' gh = \frac{1}{2}\rho v_2^2 + \rho gh$, but I am not at all comfortable with this because I feel like I am *mixing* my terms with out completely understanding why. Can anyone show me what I'm doing wrong?

Should I be adding the dynamic pressure at position 1 with the static pressure of the mercury column to start with? I expect to add the pressure of the mercury column and pressure at position 2 together because they are both located at the narrow throat that has the smaller area a, but I keep ending with the negative on the large area term A when I do that.