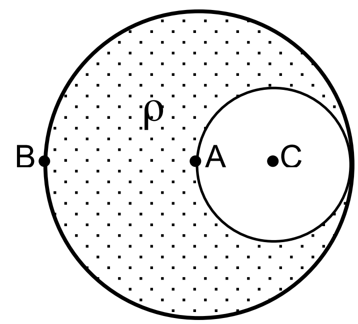
PHY 301 Practice Midterm Exam Fall 2013

Closed book (except for 1 handwritten page) - show all work

bring calculator, pen/pencil

1. [30 points] An insulating sphere of radius R, centered at point A, has uniform chare density ρ. A spherical cavity of radius $R/2$, centered at point C, is then cut out and left empty, see Fig.



(a) Find magnitude and direction of the electric field at points A and B.
(b) Find the potential at points A and B. Set $V(r \to \infty) = 0$.
(c) Write down an algebraic solution (no integrals!) for $\mathbf{E}(\mathbf{r})$ and $V(\mathbf{r})$ for the space outside the larger sphere, $r > R$. Choose $\mathbf{r} = 0$ at point A, and the radius vector of point C as $\mathbf{r} = \mathbf{R}_C$.

2. [40 points] A long solid cylindrical wire of radius a is enclosed by a coaxial conducting cylindrical shell of radius b ($b > a$, a model of a coaxial cable). A voltage can be applied between the inner wire and the outer shell (the outer shell is also called "shield").
(a) The applied voltage $\Delta V = V_b - V_a$ is such that the inner wire has a uniform linear charge density λ. Find $V_b - V_a$. Why λ is uniform?
(b) Find the capacitance (per unit length) of the cable.
(c) Find the capacitance (per unit length) if the space between the wire and the shield is partially filled with an insulator with dielectric constant $\varepsilon = 2.3$ (polyethylene), extending from $s = a$ to $s = c$, where $c < b$.
(d) Find the energy of the electric field (per unit length) for configurations in (b) and (c), U_b and U_c. Which energy, U_b or U_c is greater, if the applied voltage ΔV is the same?

3. [30 points] Two point charges $\pm q$ are located at points $\mathbf{r} = \pm \mathbf{a}$, respectively.
(a) Find the potential of this dipole $V(\mathbf{r})$. Measure the polar angle θ from the line connecting the two charges.
(b) Expand $V(\mathbf{r})$ for large distances, $r >> a$, keeping only the first nonvanishing term.
(c) Define the dipole moment $\mathbf{p} = q(2\mathbf{a})$, and find the components of the electric field $\mathbf{E}(\mathbf{r})$ of the dipole for $r >> a$.