By similar reasoning, the remaining two integrals in Eq. (16.72) are

$$\int_{PQ} + \int_{RS} = -\frac{\partial \mathcal{B}_x}{\partial y}\, dx\, dy.$$

Adding the two results, we finally get

$$\Lambda_{\mathcal{B}} = \oint_{PQRS} \boldsymbol{\mathcal{B}} \cdot d\boldsymbol{l} = \left(\frac{\partial \mathcal{B}_y}{\partial x} - \frac{\partial \mathcal{B}_x}{\partial y}\right) dx\, dy. \tag{16.73}$$

Given that dI is the current passing through $PQRS$, we may relate it to the current density $\boldsymbol{j}$ by writing

$$dI = j_z\, dS = j_z\, dx\, dy. \tag{16.74}$$

We write j_z because this is the only component of the current density $\boldsymbol{j}$ that contributes to the current dI through $PQRS$. The components j_x and j_y correspond to motions parallel to the surface and not through it. Substituting Eqs. (16.73) and (16.74) in Ampère's law, Eq. (16.67), we may write

$$\left(\frac{\partial \mathcal{B}_y}{\partial x} - \frac{\partial \mathcal{B}_x}{\partial y}\right) dx\, dy = \mu_0\, dI = \mu_0 j_z\, dx\, dy.$$

By canceling the common factor $dx\, dy$ on both sides, we obtain Ampère's law in its differential form,

$$\frac{\partial \mathcal{B}_y}{\partial x} - \frac{\partial \mathcal{B}_x}{\partial y} = \mu_0 j_z. \tag{16.75}$$

Now we can also place our surface $PQRS$ in the YZ- or the ZX-planes, resulting in the equivalent expressions

$$\frac{\partial \mathcal{B}_z}{\partial y} - \frac{\partial \mathcal{B}_y}{\partial z} = \mu_0 j_x, \tag{16.76}$$

$$\frac{\partial \mathcal{B}_x}{\partial z} - \frac{\partial \mathcal{B}_z}{\partial x} = \mu_0 j_y. \tag{16.77}$$

The three equations (16.75), (16.76), and (16.77) can be combined into one vector equation. Note that the right-hand sides are the components of the vector $\boldsymbol{j}$, the current density, multiplied by μ_0. Similarly, the left-hand sides can be considered as the component of a vector obtained from $\boldsymbol{\mathcal{B}}$ by combining derivatives in the form indicated, and it is called the *curl* of $\boldsymbol{\mathcal{B}}$, written curl $\boldsymbol{\mathcal{B}}$. Then the three equations can be consolidated in the vector equation

$$\text{curl}\, \boldsymbol{\mathcal{B}} = \mu_0 \boldsymbol{j}. \tag{16.78}$$