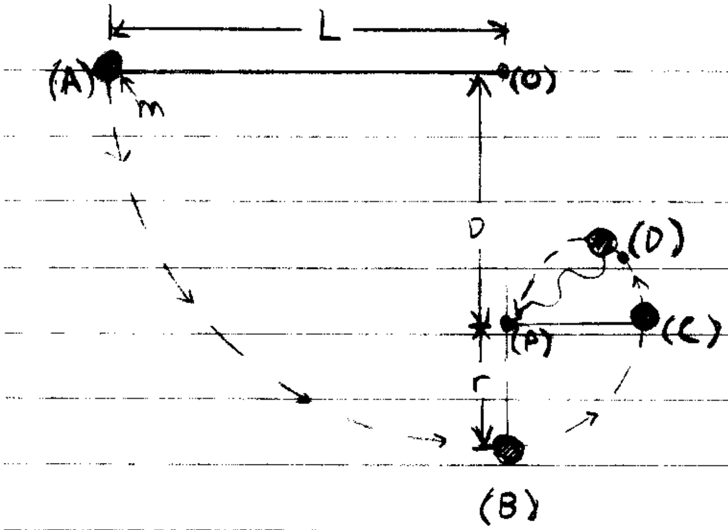


Scenario: ball of mass "m" is released from point A, tethered by string length L at rotation point O. String L swings to vertical, and catches peg P – ball swings through point C and becomes a "projectile" at some unknown angle/point D. Ball contacts point P on its descent.

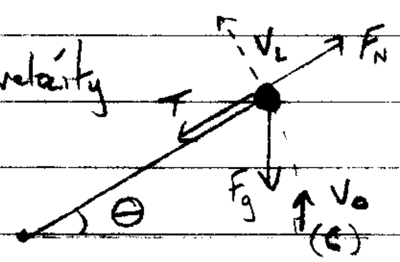
Objective: Find the necessary peg spacing (D) to allow the ball to strike point p on it's parabolic fall path.

Variables: L, r, D, some angle theta (Θ), mass (m), gravity (g).

Solution:

Part 1 – instantaneous start at point C, ball travels some angle Θ

V_L = "launch" velocity



T = Tension F_N = normal force, F_g = Force gra

$F_N = mv^2/r$ $F_g = mg \sin \Theta$

$T = F_N - F_g$; $T = mv^2/r - mg \sin \Theta$

For ball to become a projectile, $T = \emptyset$

(the ball must be INDEPENDENT of tension to be in free fall)

$T = \emptyset = mv^2/r - mg \sin \Theta$

$\cancel{m}v^2 = \cancel{m}gr \sin \Theta$, $v = \sqrt{gr \sin \Theta}$

$v = V_L$, $V_L = V_0 - \Delta V$, using WORK ENERGY THEOREM:

$(K = \frac{1}{2}mv^2,\ U = mgh)$

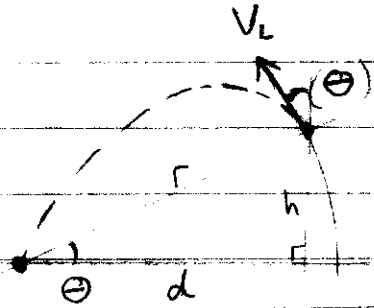
$\frac{1}{2}mV_0^2 = \frac{1}{2}mV_L^2 + mgr\sin\Theta$ $(h = r \sin \Theta)$

So, $V_0^2 = V_L^2 + 2gr\sin\theta$

$$\boxed{V_L = \sqrt{V_0^2 - 2gr\sin\theta}}$$ ← ZERO-TENSION "LAUNCH" VELOCITY

Part 2 – projectile physics angle determination

V_L ... $V_{Lx} = V_L\sin\theta$ $\quad V_{Ly} = V_L\cos\theta$



$d = r\cos\theta$ $\quad h = r\sin\theta$

pos: $S_f = S_0 + V_0 t + \frac{1}{2}at^2$

$S_f = \varnothing$, $S_0 = h$, $V_0 = V_{Ly}$, $a = -g$

$$\boxed{\frac{1}{2}gt^2 - V_{Ly}t - h = \varnothing}$$ Equation 1

pos: $s = vt$, $s = d$, $v = V_{Lx}$, $t = t$

$$\boxed{t = \frac{d}{V_{Lx}}}$$ Equation 2

$$\frac{gd^2}{2V_{Lx}^2} - \frac{V_{Ly}d}{V_{Lx}} - h = \varnothing$$ ← substitute for d, h, V_{Lx} and V_{Ly}

$$\frac{g\cancel{r^2}(\cos\theta)^2}{2V_L^2(\sin\theta)^2} - \frac{\cancel{V_L}(\cos\theta)\cancel{r}(\cos\theta)}{\cancel{V_L}(\sin\theta)} - \frac{\cancel{r}\sin\theta}{1} = \varnothing$$ ← factor out r, reduce V's

$$\frac{gr(\cos\theta)^2}{2V_L^2(\sin\theta)^{\cancel{2}}} - \frac{(\cos\theta)^2}{(\cancel{\sin\theta})} = (\sin\theta)^2$$

$$\frac{gr}{2V_L^2(\sin\theta)} - 1 = \frac{(\sin\theta)^2}{(\cos\theta)^2}$$ $\quad \frac{(\sin\theta)}{(\cos\theta)} = \tan\theta$, $\frac{(\sin\theta)^2}{(\cos\theta)^2} = \tan^2\theta$

$$\frac{gr}{2V_L^2\sin\theta} = \tan^2\theta + 1$$ ← $\tan^2\theta + 1 = \sec^2\theta$, $\sec = \frac{1}{\cos}$

$$\frac{gr}{2\sin\theta} = \frac{V_L^2}{(\cos\theta)^2},$$ Remember $T = \varnothing = \frac{\cancel{m}V^2}{r} - \cancel{m}g\sin\theta$

$V_L = \sqrt{gr\sin\theta}$

$\underline{\text{check units}}$: $g = \frac{m}{s^2}$ $r = m$, $\sin\theta = \#$

$\frac{m}{s} = \sqrt{\frac{m}{s^2} \cdot m \cdot \#}$, $\frac{m}{s} = \sqrt{\frac{m^2}{s^2}}$, $\boxed{\frac{m}{s} = \frac{m}{s}}$ ✓

so $\frac{gr}{2\sin\theta} = \frac{V_L^2}{(\cos\theta)^2}$, $V_L = \sqrt{gr\sin\theta}$ *

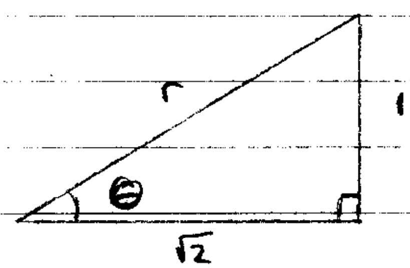
$\cancel{gr}(\cos\theta)^2 = 2(\sin\theta)(\cancel{gr}\sin\theta)$

$\frac{(\sin\theta)^2}{(\cos\theta)^2} = \frac{1}{2}$

$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{2}}$ $\frac{\sin}{\cos} = \tan$

$\boxed{\tan\theta = \frac{1}{\sqrt{2}}}$ $\longleftarrow$ ANGLE OF RELEASE / FREE-FALL!

using trig properties:



$\tan\theta = 1/\sqrt{2}$

$r = \sqrt{1^2 + \sqrt{2}^2} = \sqrt{3}$

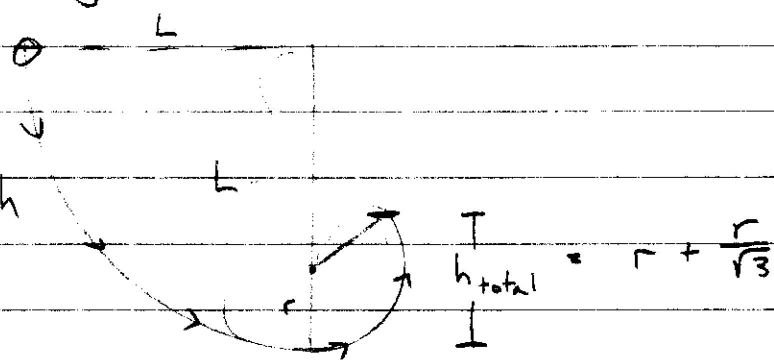
$$\boxed{\begin{aligned} \sin\theta &= \frac{1}{\sqrt{3}} = \sqrt{3}/3 \quad \text{or} \quad 1/\sqrt{3} \\ \cos\theta &= \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{6}/3 \quad \text{or} \quad \sqrt{2}/\sqrt{3} \end{aligned}}$$

$\sin\theta = 1/\sqrt{3} = h/r$, $h = r/\sqrt{3}$; overall $h = r + \frac{r}{\sqrt{3}}$

Part 3 - work-energy finalization

$h_{total} = r + \frac{r}{\sqrt{3}}$

$\frac{r\sqrt{3}}{\sqrt{3}} + \frac{r}{\sqrt{3}} = \frac{r(1+\sqrt{3})}{\sqrt{3}} = h$



$\Sigma E_0 = \Sigma E_f$

$\cancel{m}gL = \cancel{m}g\left(\frac{r(1+\sqrt{3})}{\sqrt{3}}\right) + \frac{1}{2}\cancel{m}V_L^2$ $V_L^2 = gr\sin\theta$

$\cancel{g}L = \cancel{g}\frac{r(1+\sqrt{3})}{\cancel{\sqrt{3}}} + \frac{1}{2}\cancel{g}r\frac{1}{\cancel{\sqrt{3}}}$

$L\sqrt{3} = r(1+\sqrt{3}) + \frac{1}{2}r$

$L\sqrt{3} = r + r\sqrt{3} + \frac{r}{2}$

$(L)2\sqrt{3} = 2r + 2r\sqrt{3} + r$

or $2L = \dfrac{2r(1+\sqrt{3})}{\sqrt{3}} + \dfrac{r}{\sqrt{3}}$

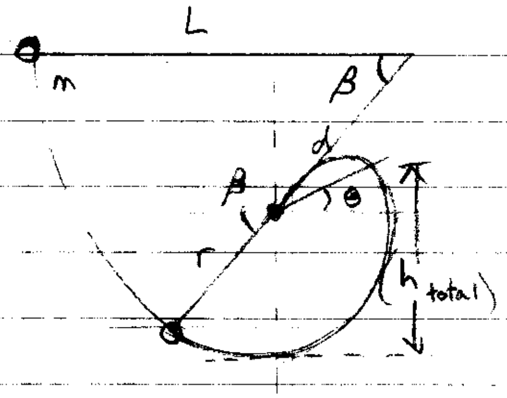
$2L = \dfrac{2r + 2r\sqrt{3} + r}{\sqrt{3}} = r(2+1+2\sqrt{3})$

$L(2\sqrt{3}) = r(3+\sqrt{12})$

$\dfrac{L}{r} = \dfrac{3+\sqrt{12}}{2\sqrt{3}} = \dfrac{\sqrt{9}+\sqrt{12}}{2\sqrt{3}} = \dfrac{\sqrt{3}+\sqrt{4}}{2} = \dfrac{2+\sqrt{3}}{2}$

$\underline{\underline{r = L\left(\dfrac{2}{2+\sqrt{3}}\right)}}$ $\longleftarrow$ Final relationship between r and L ☺

known, $r = L - d$; $d = L - r$ (... if substitution is necessary)



Scenario: Supplemental; ball of mass m is now swung on string length L to hit a peg resting at a known angle, β. Using answers obtained previously, solve for the required radius (r) as a function of the angle, β

Variables: L, r, β, m, previous angle θ (ideal release angle)

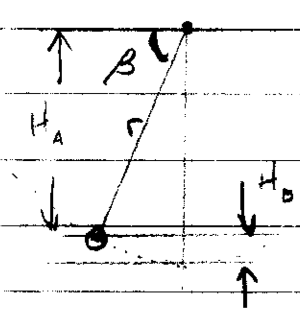
Solution: known, $\sin\theta = 1/\sqrt{3} = \sqrt{3}/3$

$$T = 0 = \frac{\cancel{m}v^2}{r} - \cancel{m}g\sin\theta \qquad V_L = \sqrt{gr\left(\frac{\sqrt{3}}{3}\right)}$$

$$h_{total} = r + \frac{r}{\sqrt{3}}$$

Part 1 - original height above bottom of swing

$$H_o = L\sin\beta + H_B, \qquad \underline{\underline{H_o = L\sin\beta + r(1-\cos\beta)}}$$



$$H_B = r - r\cos\beta$$

Part 2 - work energy theorem

$$mgH_o = mgh_{total} + \frac{1}{2}mV_L^2$$

$$H_o = L\sin\beta + r(1-\cos\beta)$$

$$h_{total} = r + r/\sqrt{3}$$

$$V_L = \sqrt{gr/\sqrt{3}}$$

$$\cancel{m}\cancel{g} H_0 = \cancel{m}\cancel{g} h_{total} + \frac{1}{2}\cancel{m} V_L^2 / g$$

$$H_0 = h_{total} + \frac{V_L^2}{2g}$$

$$L \sin\beta + r(1 - \cos\beta) = r + r/\sqrt{3} + r/2\sqrt{3}$$

$$L \sin\beta = r\left(\cancel{1} + \frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{3}} - \cancel{1} + \cos\beta\right)$$

$$\boxed{r(\beta) = \frac{(L \sin\beta)}{\left(1/\sqrt{3} + 1/2\sqrt{3} + \cos\beta\right)}} \leftarrow$$ r as a function of β, L constant

$r = L - d$, $d = L - r$ if you need to substitute