[DRAFT] Towards a Better Understanding of Lift and Drag: Deflection Theory

Abstract

"Deflection Theory" is an application of Newton's Third Law to the interaction between airfoils and freestream flow that physically couples lift and drag, even in incompressible and inviscid flow. The current consensus is that airfoils in incompressible and inviscid flow cannot produce drag. Deflection Theory introduces the concept of modeling the change in direction of the portion of the freestream that interacts with the airfoil. NACA 0009 experimental data corroborate lift, drag, and lift-to-drag ratio predictions.

I. Introduction

Current airfoil performance models for incompressible and inviscid flow predict lift without drag. Conservation of energy dictates that airfoil forces cannot be parallel to freestream flow at the airfoil, and this is the reason for d'Alembert's Paradox. Aerodynamic forces in inviscid and incompressible potential flow are therefore normal to the flow direction at the airfoil , and drag comes from this change in flow direction. In the three-dimensional context, trailing wake vortex modeling tilts flow at the airfoil to cause "lift-induced drag". However, no well-known direct relationship between lift and drag exists for the purely two-dimensional context. Deflection Theory offers an explanation for two-dimensional "lift-coupled drag".

Thin Airfoil Theory and the Vortex Panel Method assume a fixed freestream flow, independent of any forces exerted on the airfoil. By the conservation of momentum, freestream flow downstream of the airfoil should be deflected to oppose the resultant aerodynamic forces on the airfoil. Dragless lift is a valid transient solution, but not a steady-state one. Deflection Theory is able to predict drag as naturally as lift because the conservation of momentum is applied to the portion of freestream flow that interacts with the airfoil.

The common lift-producing spinning cylinder example clearly shows the deficiency of current lift and drag analysis methods. Everyone would agree that generally speaking, an airfoil produces lift by "pushing down" air, and this could be said of any object that produces lift. However, the traditional spinning cylinder solution has a flow field with velocity magnitudes and streamlines exactly symmetrical from fore and aft of the cylinder. This clearly implies that just as much air is pulled up as it is pushed down. This clearly violates Newton's Third Law.

II. Methods

Newton's Third Law dictates that air must be deflected in opposition to airfoil lift force. It will be shown that such deflection also causes drag, which means drag is coupled with lift. Lift without drag therefore voilates Newton's Third Law.

Suppose the portion of the freestream flow that interacts with the airfoil has a mass flow rate of $\dot{m}$ and a velocity V_∞. If we take the average angle of deflection of the airflow to be ϕ, lift (L) and drag (D) can be calculated as:

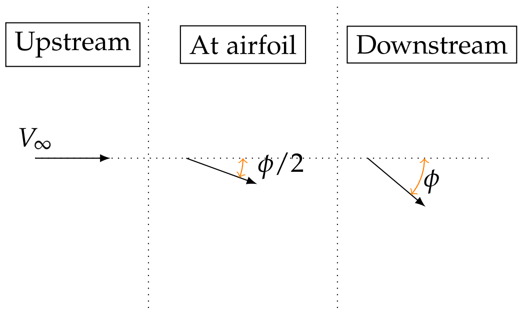

Figure 1: *Vector illustration flow direction changes (angles exaggerated). Vector magnitudes remain the same.*

$$L = \dot{m}V_\infty sin(\phi) \tag{1}$$

$$D = \dot{m}V_\infty(1 - cos(\phi)) \tag{2}$$

$$\frac{L}{D} = \frac{sin(\phi)}{1 - cos(\phi)} \tag{3}$$

Since airfoils in incompressible and inviscid flow can not produce forces parallel to the flow at the airfoil, the relationship between the induced flow angle (γ) and the lift-to-drag ratio is:

$$\frac{L}{D} = \frac{cos(\gamma)}{sin(\gamma)} \tag{4}$$

Equating the two expressions for lift-to-drag ratio:

$$\frac{sin(\phi)}{1 - cos(\phi)} = \frac{cos(\gamma)}{sin(\gamma)} \tag{5}$$

Through trigometric identities:

$$\gamma = \frac{\phi}{2} \tag{6}$$

The airfoil "sees" the freestream tilted at an angle exactly half that of the full deflection angle downstream of the airfoil.

To relate C_N and the deflection angle ϕ:

$$F = \sqrt{L^2 + D^2} \tag{7}$$

Where A_∞ is the mass flux area of the freestream that interacts with the airfoil:

$$\dot{m} = \rho A_\infty V_\infty \tag{8}$$

Substituting (1), (2), and (8) into (7):

$$F = \rho A_\infty V_\infty^2 \sqrt{2 - 2cos(\phi)} \tag{9}$$

Normalizing F to C_N:

$$C_N = \frac{A_\infty}{A_w}\sqrt{8 - 8cos(\phi)} \tag{10}$$

The ratio A_∞/A_w, where A_w is the reference area of the wing, is the final loose end of Deflection Theory. Unfortunately at present I do not have a physical way to estimate this parameter. A_∞/A_w may be constant, or dependent on the force exerted on the airfoil, or something else. Presently the ratio is chosen arbitrarily as to reasonably match the experimental data to demonstrate Deflection Theory's potential.

We can utilize existing performance prediction methods such as Thin Airfoil Theory and Vortex Panel Method by simply taking their results as the normal force coefficient C_N, instead of, as traditionally assumed, the lift coefficient C_L. For simplicity, we will use Thin Airfoil Theory to obtain a relationship between angle of attack (α) and C_N:

$$C_N = 2\pi\alpha \tag{11}$$

We must make the distinction between the aerodynamic angle of attack (α) and the global angle of attack (α_g):

$$\alpha_g = \gamma + \alpha \tag{12}$$

NACA 0009 airfoil data were chosen for comparison, because it is a thin and symmetric airfoil and has publicly available performance data. The minimum drag coefficient from experimental data was added to the Deflection Theory model to account for skin friction and baseline unsteady drag.

III. Results

Figures 2, 3, and 4 demonstrate Deflection Theory's efficacy up until stall.

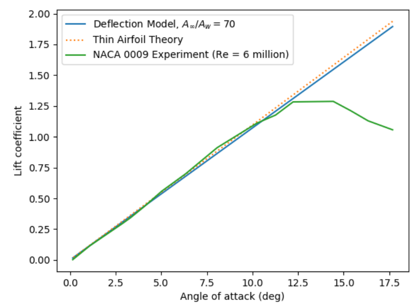

Figure 2: *Lift coefficient predictions from Thin Airfoil Theory, the present Deflection Theory, and NACA 0009 experimental data.*

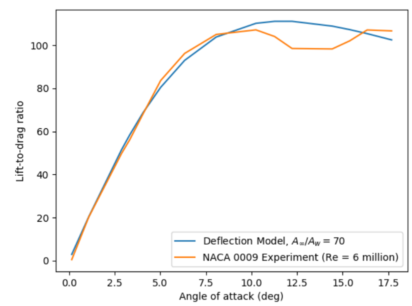

Figure 4: *Lift-to-drag ratio predictions from the present Deflection Theory, and NACA 0009 experimental data.*

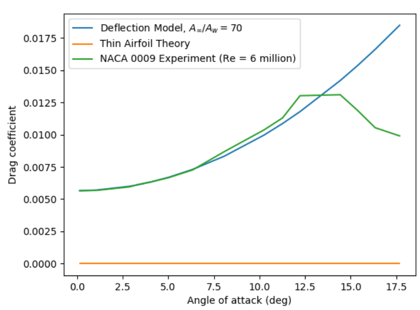

Figure 3: *Drag coefficient predictions from Thin Airfoil Theory, the present Deflection Theory, and NACA 0009 experimental data.*

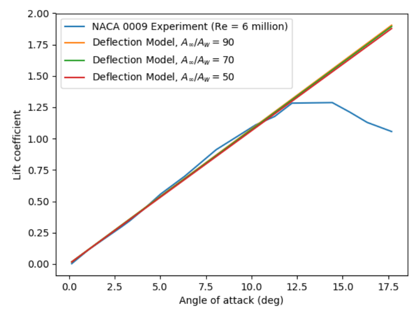

Figure 5: *Sensitivity of Deflection Theory lift predictions to the A_∞/A_w parameter.*

Figures 5, 6, and 7 show the sensitivity of performance on A_∞/A_w. Lift predictions are much less sensitive to A_∞/A_w than the drag predictions are.

IV. Discussion

Deflection Theory is a novel physical argument somewhat corroborated by experimental data and provides a possible way to better understand lift and drag. It is important to remember that for a complete performance prediction model, Deflection Theory requires another method to compute C_N. In this paper, Deflection Theory was used to enhance Thin Airfoil Theory. The same can be easily done for Vortex Panel Methods.

Deflection Theory explains drag produced by unsteady means, including when no net lift is generated. Freestream deflection oscillation such that the mean lift is zero will result in a non-zero drag. Similarly, simultaneous steady opposing flow deflections can also produce non-zero drag without net lift.

It was expected for the model derived in this paper to match well with experiment until stall, because Thin Airfoil Theory was used to compute C_N. A stalled airfoil is not an effective deflector of the freestream flow, but the Deflection Theory still holds. Computing the C_N is not the responsibility of Deflection Theory.

Currently the lack of a method to determine the extent of the freestream flow that interacts with the airfoil, represented by the parameter A_∞/A_w, is the most glaring deficiency. However, it might be reasonable to empirically determine an estimate for a set of similar airfoils.

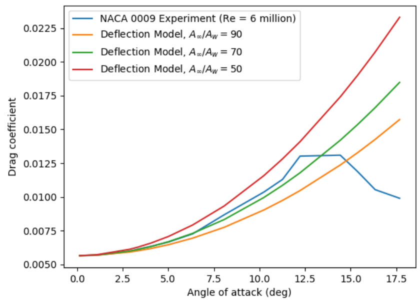

Figure 6: *Sensitivity of Deflection Theory drag predictions to the A_∞ / A_w parameter.*

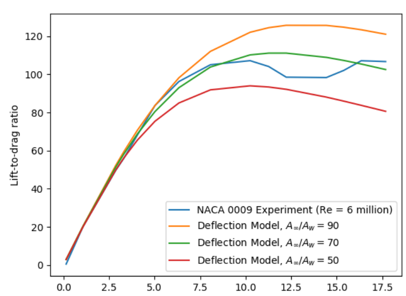

Figure 7: *Sensitivity of Deflection Theory lift-to-drag ratio predictions to the A_∞ / A_w parameter.*