GFauxPas
P52 Prob 6

We are given various assumptions about the nature of a theoretical PC manufacturer. We are asked to analyze the data and assumptions given in order to make inferences about the best choices for the company to make to maximize profit. We analyze these choices.

Assumptions and definitions:

The base selling price for a PC is \$950. However, we are interested in possibly lowering the price in order to increase revenue. Thus,

$price := 950 - 100 \cdot d$

$$950 - 100\, d \tag{1}$$

The current costs incurred are \$700 per computer produced. So far, \$50 000 is the monthly budget for advertising costs, but we are considering raising the advertising budget in order to increase sales. So,

$cost := 700 \cdot units + 50000 + 10000 \cdot a$

$$700\,(10000. + 200\,a)\left(1 + \frac{1}{2}\,d\right) + 50000 + 10000\,a \tag{2}$$

Baseline, 10 000 units are sold a month. We assume a decrease in price by \$100 increases units sold by 50%. We also assume that each increase in advertising budget by 10 000 will increase units sold by 200. Assuming all dependencies are linear:

$units := (1e4 + 200 \cdot a) \cdot \left(1 + \frac{1}{2} \cdot d\right)$

$$(10000. + 200\,a)\left(1 + \frac{1}{2}\,d\right) \tag{3}$$

We are told that the maximum advertising budget is \$100 000 a month, so:

$0 \le a \le 5$

$$0 \le a \textbf{ and } a \le 5 \tag{4}$$

and of course we cannot give the computers away, so the price must be positive:

$950 - 100 \cdot d > 0$

$$0 < 950 - 100\, d \tag{5}$$

Summarizing:

$units := (1e4 + 200 \cdot a) \cdot \left(1 + \frac{1}{2} \cdot d\right)$
$cost := 700 \cdot units + 50000 + 10000 \cdot a$
$price := 950 - 100 \cdot d$
$revenue := price \cdot units$
$profit := revenue - cost$

$$(950 - 100\,d)\,(10000. + 200\,a)\left(1 + \frac{1}{2}\,d\right) - 700\,(10000. + 200\,a)\left(1 + \frac{1}{2}\,d\right) - 50000 - 10000\,a \tag{6}$$

The feasible region will be defined on the *ad*-plane by $a = 0$, $a = 5$, $d < 950/100$.

We find the critical points of *profit*:

$solve(\{diff(profit, d) = 0, diff(profit, a) = 0\}, [a, d])$

$$[[a = -50., d = 2.265564437], [a = -50., d = -1.765564437]] \quad \textbf{(7)}$$

But these lie outside the feasible region.

We try on the curve a = 0

$solve(\{diff(profit, d) = \text{lambda}\cdot diff(a, d), diff(profit, a) = \text{lambda}\cdot diff(a, a), a = 0\})$

$$\{a = 0., d = 0.2500000000, \lambda = 40625.\} \quad \textbf{(8)}$$

$subs\left(a = 0, d = \frac{1}{4}, profit\right)$

$$2.481250000\ 10^6 \quad \textbf{(9)}$$

This might be it.

We try on the curve a = 5:

$solve(\{diff(profit, d) = \text{lambda}\cdot diff(a, d), diff(profit, a) = \text{lambda}\cdot diff(a, a), a = 5\})$

$$\{a = 5., d = 0.2500000000, \lambda = 40625.\} \quad \textbf{(10)}$$

$subs\left(a = 5, d = \frac{1}{4}, profit\right)$

$$2.684375000\ 10^6 \quad \textbf{(11)}$$

That's even better.

We check at the "corner points", though:

$subs\left(a = 0, d = \frac{950}{100}, profit\right)$

$$-4.030000000\ 10^7 \quad \textbf{(12)}$$

$subs\left(a = 5, d = \frac{950}{100}, profit\right)$

$$-4.437500000\ 10^7 \quad \textbf{(13)}$$

Those definitely aren't it.

Our best choice is then the point (*a,d*) = (5,1/4), at which the profit is ~2.684e6.
That is, one should increase the investment to the full (50 000 + 10 000 * 5) = $100 000.
Also, one should set the selling price equal to (950 - 25) = $925.