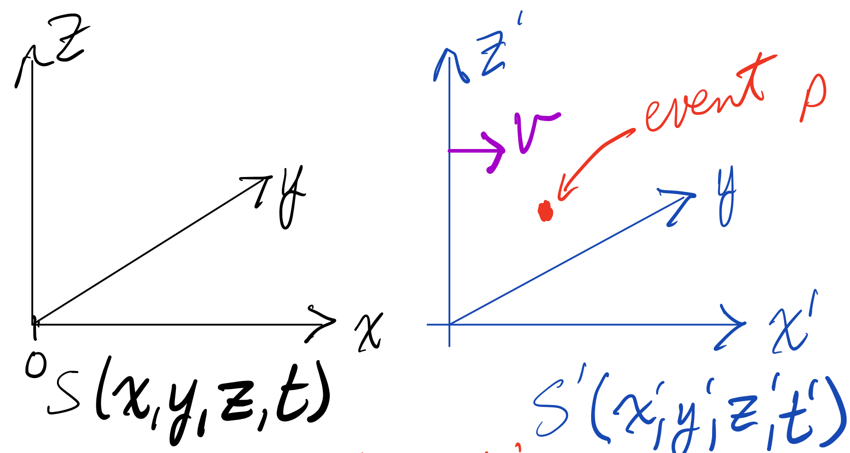


galilean transformation: $x' = x - vt$

$y' = y$

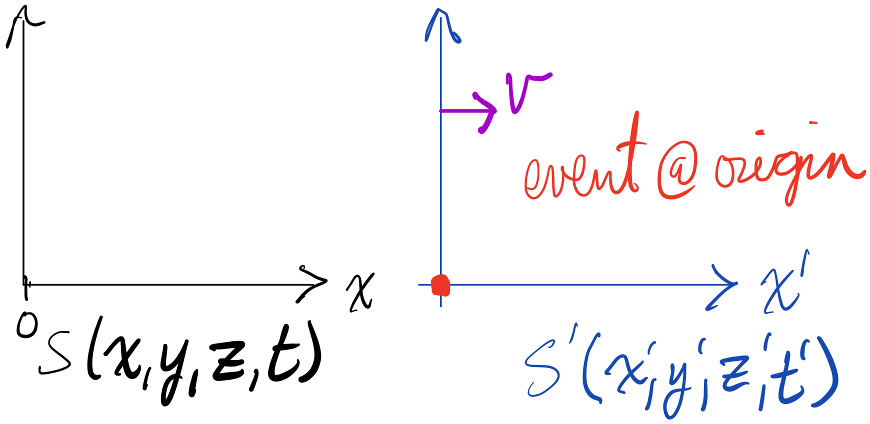
$z' = z$

$t' = t$

- Lorentz transformation between S + S' must be one-to-one, or linear.
- Linear equations follow the form $\boxed{y = ax + bt}$ where a and b are constant.

Assume an event @ origin of S' and time $t' = 0$ with both S and S' starting @ $x = x' = 0$ and $t = t' = 0$

As S' moves w/ velocity V in the x direction, the light propagates a distance of $x' = ct'$ and $x = ct$



using eq.①: $x' = ax + bt$, $x' = 0$

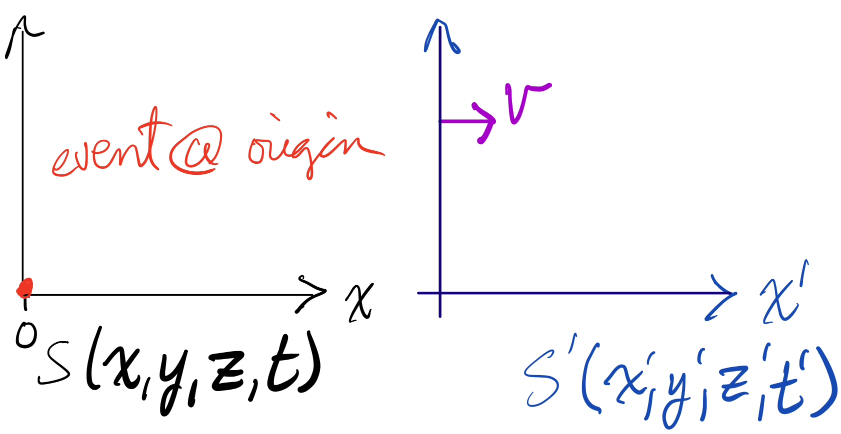
from S', $x = vt$ and $0 = avt + bt$

$av\cancel{t} = -b\cancel{t} \longrightarrow b = -av$

sub in to eq.①: $x' = ax - avt$

$x' = a(x - vt)$

solve the reverse transformation



$x = ax' + bt'$, $x = 0$. from S origin $x' = -vt'$ and $0 = -avt' + bt'$

$avt' = bt' \Rightarrow b = av$ so $x = ax' + avt' = a(x' + vt')$

L Given the postulate that the speed of light is constant in all inertial frames: $x = ct$ and $x' = ct'$

using the highlighted equations above:

$$ct = a(ct' + vt') \quad \text{AND} \quad ct' = a(ct - vt)$$

$$t/t' = \frac{a(c+v)}{c} \qquad \frac{c}{a(c-v)} = t/t'$$

$$\therefore \quad \frac{a(c+v)}{c} = \frac{c}{a(c-v)}$$

$$a^2(c+v)(c-v) = c^2$$

$$a^2(c^2 - v^2) = c^2 \longrightarrow a^2 = c^2/(c^2 - v^2) = \frac{c^2}{c^2} \frac{1}{(1 - v^2/c^2)}$$

$$a = \sqrt{\frac{1}{1 - v^2/c^2}} \equiv \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Now with the constant "a" known, which we call γ we can solve for t and t' transformations by using the highlighted equations above.

$$x' = \gamma(x - vt) \quad \text{AND} \quad x = \gamma(x' + vt')$$

$$\frac{x}{\gamma} = \gamma(x - vt) + vt' \Longrightarrow x\sqrt{1 - v^2/c^2} = \frac{x - vt}{\sqrt{1 - v^2/c^2}} + vt'$$

$$= \quad x\sqrt{1 - v^2/c^2} = \frac{x - vt + vt'\sqrt{1 - v^2/c^2}}{\sqrt{1 - v^2/c^2}}$$

$$= \quad x(1 - v^2/c^2) = x - vt + vt'\sqrt{1 - v^2/c^2}$$

$$= \quad x - \frac{xv^2}{c^2} = x - vt + vt'\sqrt{1 - v^2/c^2}$$

$$= -\frac{xv^2}{c^2} = -vt + vt'\sqrt{1-v^2/c^2}$$

$$= \frac{xv}{c^2} = t - t'\sqrt{1-v^2/c^2}$$

$$= t'\sqrt{1-v^2/c^2} = t - \frac{xv}{c^2}$$

$$= t' = \gamma\left(t - \frac{xv}{c^2}\right)$$

replace v with -v for reverse transformation

$$t = \gamma\left(t' + \frac{x'v}{c^2}\right)$$

Given $x' = \gamma(x - vt)$ and $t' = \gamma\left(t - \frac{xv}{c^2}\right)$

$dx' = \gamma(dx - v\,dt)$ and $dt' = \gamma(dt - \frac{v}{c^2}dx)$

velocity $u'_x = \frac{dx'}{dt'} = \frac{\gamma(dx - v\,dt)}{\gamma(dt - (v/c^2)dx)} = \frac{dx - v\,dt}{dt\left(1 - \frac{v}{c^2}\frac{dx}{dt}\right)}$

$$= \frac{\frac{dx}{dt} - v\left(\frac{dt}{dt}\right)}{1 - \frac{v}{c^2}\frac{dx}{dt}}$$

let $u_x = \frac{dx}{dt}$ then

$$u'_x = \frac{u_x - v}{1 - \frac{v u_x}{c^2}}$$

replace v with -v for reverse transformation

$$u_x = \frac{u'_x + v}{1 + \frac{u_x v}{c^2}}$$