Euclid's 5th postulate, as written in The Elements, is saying too much.
I believe what he wrote is better labeled a theorem.

A better 5th postulate might go like this:

My 5th: If a straight line falling on two straight lines makes a pair of corresponding angles unequal, then the two straight lines are not parallel.

In addition, there are a couple of propositions I would add:

Proposition A (added, does not use 5th)
If a straight line falling on two straight lines makes the sum of the interior angles on the same side less than two right angles, then the sum of the interior angles on the opposite side will be greater than two right angles.

Proposition B (added, does not use 5th)
If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines will not meet on the opposite side.

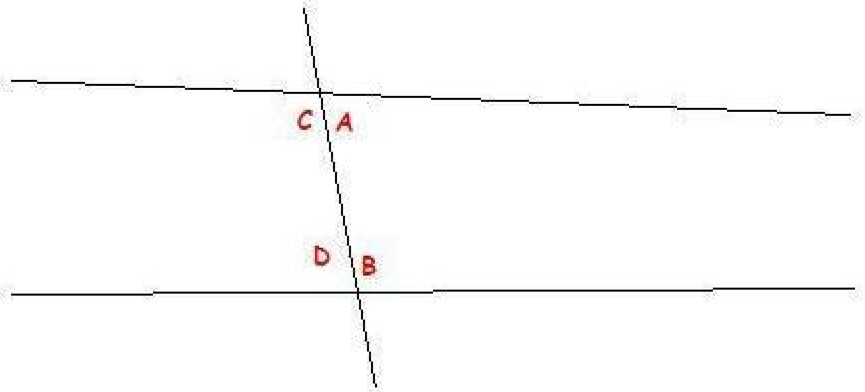
To prove Proposition A, you can use the fact that adjacent angles on a straight line are supplementary.

To prove Proposition B, you can use the exterior angle theorem.

Euclid's original 5th postulate can then be labeled Proposition C.
You can use Proposition A, Proposition B and My 5th to prove it.

Proposition C (was Euclid's 5th)
If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

PROPOSITION A



GIVEN: A+B<180

PROVE: C+D>180

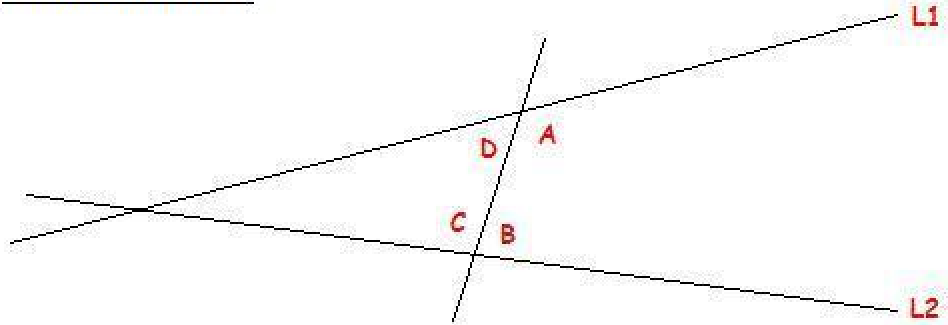
PROOF:

A=180-C AND B=180-D <adjacent angles on staight line>

THEN (180-C)+(180-D)<180 <substitution>

THEN C+D>180

PROPOSITION B



GIVEN: A+B<180

PROVE: L1 and L2 will not meet on the opposite side.

PROOF: Assume, for the sake of argument, that L1 and L2 meet on the opposite side. Then by the exterior angle theorem, A>C and B>D. Adding, we get : A+B>C+D

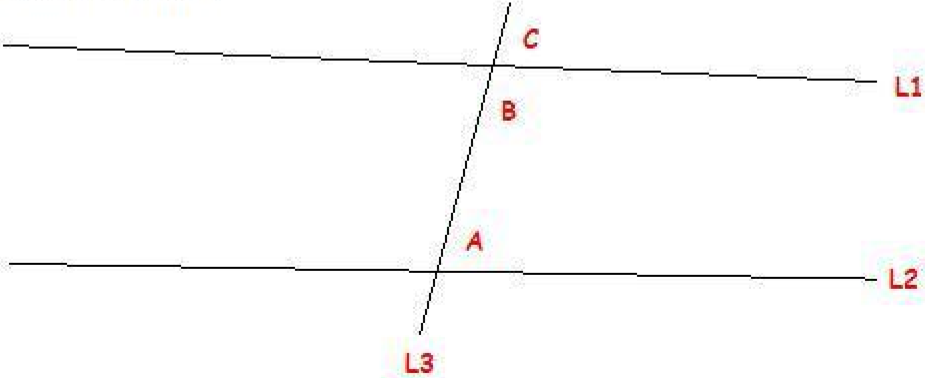
But A+B<180 <given>

Then D+C<180.

But this is impossible, because by PROPOSITION A, D+C>180.

Hence, if A+B<180, L1 and L2 will not meet on the opposite side.

PROPOSITION C



GIVEN: L3 crosses L1 and L2 making A+B<180.

PROVE: L1 and L2 will intersect on the side where A+B<180.

PROOF: B=180-*C* <adjacent angles on straight line>

But A+B<180 <given>

Then A+(180-*C*)<180 <substitution>

Then A<C

Then L1 and L2 must intersect by *My Fifth*, and my Proposition B tells you it can only be on the side where the sum of the interior angles is less than two right angles, which identifies this side by Proposition A.