Challenges to Faraday's flux rule

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Faraday's law (or flux rule) is beautiful in its simplicity, but difficulties are often encountered when applying it to specific situations, particularly those where points making contact to extended conductors move over finite time intervals. These difficulties have led some to challenge the generality of the flux rule. The challenges are usually coupled with the claim that the Lorentz force law is general, even though proofs have been given of the equivalence of the two for calculating instantaneous emfs in well-defined filamentary circuits. I review a rule for applying Faraday's law, which says that the circuit at any instant must be fixed in a conducting material and must change continuously. The rule still leaves several choices for choosing the circuit. To explicate the rule, it will be applied to several challenges, including one by Feynman. © *2004 American Association of Physics Teachers.*

I. INTRODUCTION

Scanlon and Henriksen have commented that "It is interesting that Faraday's electromagnetic induction law, despite its widespread technological applications, continues to present challenges in its interpretation."[1] This law can be expressed as

$$\mathcal{E}_{\text{flux}} = -\frac{d\Phi}{dt}, \tag{1}$$

where $\mathcal{E}_{\text{flux}}$ is the induced emf around a circuit, Φ is the magnetic flux through the circuit, and the circuit may be moving or at rest. Equation (1) is appealing because it yields the emf as the time derivative of a single quantity, Φ.

Challenges to the flux rule continue,[2–6] and in response, I will revisit the work of Ref. 7 in which various examples of electromagnetic induction that have acquired reputations as "exceptions to the flux rule" were considered. Reference 7, which carefully examined relativistic aspects of Faraday's law and various nonrelativistic challenges, is required reading for anyone who wants to thoroughly understand electromagnetic induction.

If a circuit moves in a magnetic field that is constant in time, the mechanism causing the induced emf arises from the motional part of the Lorentz force per unit charge, $\mathbf{v} \times \mathbf{B}$. Because the general case involves the total Lorentz force per unit charge, $\mathbf{E} + \mathbf{v} \times \mathbf{B}$, the integral of this force around a contour will be referred to as the "Lorentz" or "$\mathbf{v} \times \mathbf{B}$" rule for calculating the emf, and the resulting value will be designated $\mathcal{E}_{\text{motion}}$, because the cases we will consider involve $\mathbf{v} \times \mathbf{B}$. The beautiful fact discovered by Faraday is that a single law, Eq. (1), encompasses both $\mathbf{E}$ and $\mathbf{v} \times \mathbf{B}$ mechanisms.

For a well-defined filamentary (that is, thin or wire-like) circuit, the equivalence of Eq. (1) and the Lorentz rule for instantaneous emfs can be shown by transforming the closed line integral of the two parts of the Lorentz force into separate rates of change of flux due to motion and changing field, as shown in Refs. 8–11. All of these proofs of equivalence use the fact that $\mathbf{B} \cdot (\mathbf{v} \times d\mathbf{s})$ is the rate of change of the flux through the area $\mathbf{v} \times d\mathbf{s}$ swept out by a conductor length element $d\mathbf{s}$. (With this convention for the area element, the direction of the induced $\mathbf{E}$-field is parallel to $d\mathbf{s}$ if $d\Phi$ is

negative and antiparallel to $d\mathbf{s}$ if $d\Phi$ is positive.) But there are situations where the circuit is not so well-defined over a finite time interval, and that is where many of the supposed exceptions to Faraday's law arise. Particularly troublesome are situations involving extended conductors or moving contact points. Challengers[2–6,12–14] claim that the Lorentz rule trumps Faraday's law in such circumstances. A commonly cited example is the Faraday disk, shown in Fig. 1. It and other systems, such as the unipolar generator (where the field source, a magnet, also forms part of the circuit), have been offered as challenges (Refs. 2–6, 12–14). Feynman[14] also presented a challenging example that has been widely cited.[15–17]

Several of the more interesting challenges, including Feynman's, were considered only briefly in Ref. 7. These authors emphasized the importance of the circuit being fixed in the conducting material, and that changes in the circuit be continuous. This prescription still leaves several choices for defining a circuit over time. To clarify matters, I will use the Faraday disk in Sec. II to illustrate a procedure for handling such situations, in particular those involving difficulties with moving contact points. The procedure conforms to the "continuous transformation" prescription of Ref. 7, but is a bit more specific. The procedure will then be applied in Sec. III to the unipolar generator and in Sec. IV to Feynman's challenge. Section V offers a few conclusions.

II. THE LESSON OF THE FARADAY DISK

Consider the Faraday disk[18] shown in Fig. 1, where a metal disk of radius a rotates in a constant magnetic field that is directed perpendicular to the entire circular area of the disk. The disk completes a circuit, with one contact point on the axis of the disk and the other on the rim. Griffiths[4] has written that the flux rule (Faraday's law) is not useful here because the current is spread throughout the entire disk, that is, there is no well-defined filamentary path. He advocated going directly to the Lorentz force law (the $\mathbf{v} \times \mathbf{B}$ term in this case), but his objection to the flux rule regarding the lack of a well-defined path would, if valid, apply here too. Nevertheless, he follows conventional practice and chooses the simplest path, a radius fixed in the rotating disk. Several
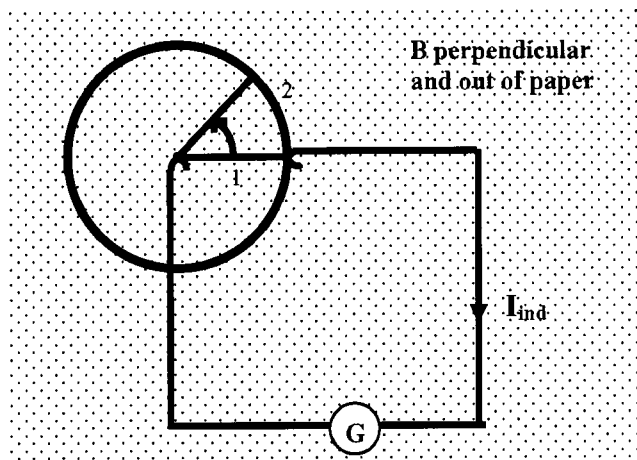
Fig. 1. The lesson of the homopolar generator. At $t=0$, the circuit containing the galvanometer is completed by material segment 1 fixed in the rotating disk. For $t>0$, the circuit is completed by this material segment plus material segment 2, consisting of the portion of the rim along which the contact point moved.
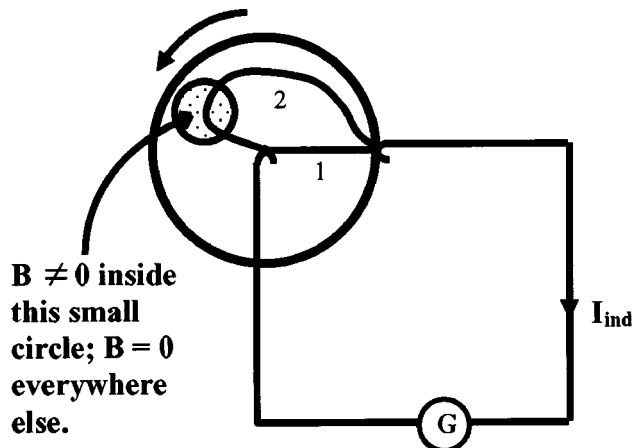

Fig. 2. An illustration of the ambiguity of the Lorentz force calculation, which gives a zero result when the circuit is completed by path 1, but a nonzero result when completed by path 2. The ambiguity arises because the line integral of $\mathbf{v}\times\mathbf{B}$ and the flux swept out are both path-dependent.

authors have chosen the same path and successfully applied Faraday's law to the Faraday disk.[19–21] So either approach appears to work here.

Figure 1 shows how Faraday's law can be applied to the Faraday disk (also known as the homopolar generator). As mentioned, a convenient filamentary path in the disk consists of a radius fixed in the rotating disk. When the calculation is made over a finite time, the circuit is completed by an arc of rim material from the end of the radius to the nonrotating contact point of the stationary lead wire on the rim. We can think of this arc as being generated by the motion of the contact point relative to the disk. (A more complicated application of this idea is provided in Ref. 22 for a unipolar generator.) The area swept out is $a^2\theta/2$ for angle $\theta=\omega t$, so the emf is

$$\mathcal{E}_{\text{flux}}=\frac{Ba^2\omega}{2},\tag{2}$$

which is the standard result given by a $\mathbf{v}\times\mathbf{B}$ calculation applied to the rotating radius.

The key point of the flux solution of the homopolar generator is that the material associated with the path traced out on the rim by the moving contact point is added to the original material (the radius fixed in the rotating disk) that completes the circuit at $t=0$. In this case, the added material does not sweep out any area, but in other situations, for example, Feynman's,[14] the added material does sweep out an area. The generation of added path material by moving contact points leads to the following procedure for applying Faraday's law. When calculating the emf around a circuit, the circuit must consist of material that is always instantaneously fixed in the conductor and is modified only by continuously adding or removing material instantaneously fixed in the conductor. In many cases, it is easiest to keep track of the flux through areas required for Faraday's law by retaining material needed at $t=0$ and adjusting it continuously to complete the circuit at later times. The significance of the word "instantaneous" will be clarified when Feynman's challenge is considered.

The typical treatment of the Faraday disk assumes the $\mathbf{B}$-field is constant across and perpendicular to the circular area of a solid conducting disk. For rigid body rotation, the conditions $\mathbf{v}=\boldsymbol{\omega}\times\mathbf{r}$ and $\boldsymbol{\omega}\times\mathbf{B}=0$ imply that $\nabla\times(\mathbf{v}\times\mathbf{B})=0$. Therefore, the integral of $\mathbf{v}\times\mathbf{B}$ along a path from the center to a fixed rim point, and the area swept out by this same path, are independent of the path and yield a result consistent with experiment. This path independence is in a nutshell why both the flux and $\mathbf{v}\times\mathbf{B}$ methods work. More generally, a uniform $\mathbf{B}$-field allows for great freedom in choosing a circuit path when dealing with extended conductors.

To see a situation where both standard approaches fail, consider the situation in Fig. 2, where the field is not constant, and it is concentrated over a limited off-center region of the disk. The application of $\mathbf{v}\times\mathbf{B}$ to paths that avoid this region yield zero (or very tiny) emfs, while paths threading through the region give nonzero results. The same ambiguity affects flux calculations. This ambiguity does not preclude a well-defined calculation of the emf by flux or $\mathbf{v}\times\mathbf{B}$ methods along an arbitrary path, but the result is path-dependent, and it is not clear how useful such a result would be. (Perhaps an average over all paths fixed in the disk between center and a fixed point at the circumference would yield the emf actually measured.) In any case, neither method seems to work unambiguously, which casts doubt on the supposed greater generality of the $\mathbf{v}\times\mathbf{B}$ method.

III. THE UNIPOLAR GENERATOR

As an application of the procedure discussed in Sec. II, consider the unipolar generator shown in Fig. 3.[23] Here, we have a permanent magnet and an open wire structure consisting of four straight segments: a vertical one starting at the top center of the magnet, a horizontal one at the top, a vertical one down the left side, and another shorter bottom horizontal segment going to the side surface of the magnet. As shown in Fig. 3, the structure is rotating and the magnet is fixed. The circuit at $t=0$ consists of the four segments just mentioned, plus the stationary radius from the side contact point to the axis of the magnet and the magnet axis going up to the other contact point of the wire structure. When the structure has turned through the angle θ, the circuit again consists of the four segments, and reminiscent of the homopolar generator, is completed by a circular arc on the magnet rim back to the original radius which returns to the axis.
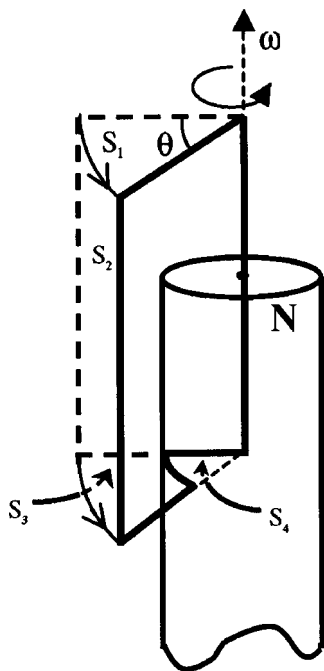
Fig. 3. The unipolar generator, shown here with the four-segment wire rotating about the axis of the magnet.
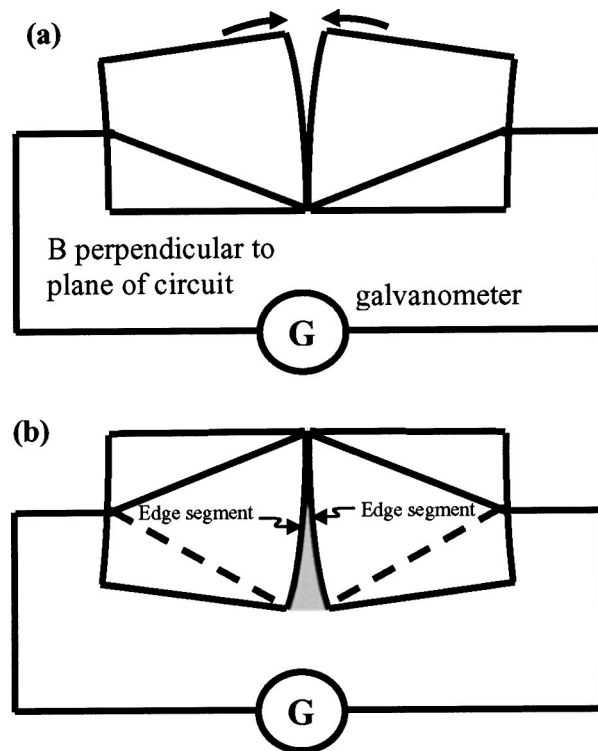

Fig. 4. Feynman's challenge. (a) Before rocking; (b) after rocking. Feynman correctly concludes that the induced emf should be very small, but incorrectly concludes that the "flux rule" fails because his incorrect choice of path after rocking increases the enclosed flux too much (Ref. 14, p. 17-3). The shaded area shows the small area swept out by the edge faces used to complete the path if the original path in part (a) is retained. The original path similarly sweeps out a small area during the rotation, so properly applied, Faraday's law also leads to a small emf.

The top, left-side, and bottom structure segments move through a **B**-field and calculation by $\mathbf{v}\times\mathbf{B}$ clearly shows that there must be an emf. The challenge to the flux rule originates from the fact that the field has no azimuthal component, so the flux through the vertical surface defined by the wire structure is zero at all times. Supposedly, the absence of flux means there is no induced emf, which contradicts the $\mathbf{v}\times\mathbf{B}$ result. But calculating flux through this vertical surface is a clear misunderstanding of Faraday's law, because the four segments of the wire structure by themselves do not define a complete circuit through which flux changes can be determined.

One way to approach the issue is to look at the areas swept out by the moving segments. Consider the flux through the three swept-out areas S_1, S_2, and S_3 generated when the wire structure swings through the angle $\theta = \omega t$: S_1, generated by the top horizontal segment, is pie-shaped; S_2 is a side area segment of a cylinder; and S_3, generated by the bottom segment of the structure, is the outer portion of a pie-shaped segment. The inner portion of the pie, inside the magnet, is shown as S_4. Because the total flux through the closed surface $S_1 + S_2 + S_3 + S_4$ is zero, the magnitude of the flux through $S_1 + S_2 + S_3$ is equal to the magnitude through S_4. Indeed, S_4 is the *horizontal* flux-capturing area inside the closed path after rotation starts, which now includes the arc segment on the magnet surface traced out by the moving contact point. This closed path is shown by the heavy lines in Fig. 3 and is the appropriate closed path to use for the flux rule. The appeal of the "swept out" picture is the connection it makes to the material experiencing $\mathbf{v}\times\mathbf{B}$ forces.

To make an actual calculation of emf, assume, for simplicity, that **B** inside the magnet is constant across S_4, so $\Phi_4 = Ba^2\theta/2$. (This assumption will be very good if the bottom part of the circuit is very far from either end of the magnet.) Therefore, by the flux method, the induced emf is

$$\mathcal{E}_{\text{flux}} = \frac{Ba^2\omega}{2}. \tag{3}$$

$\mathcal{E}_{\text{motion}}$ would be difficult to calculate because **B** in the space where the conductors (segments of the wire structure) are moving does not necessarily have a simple mathematical form. But the equality of $\mathcal{E}_{\text{motion}}$ and $\mathcal{E}_{\text{flux}}$ is assured by the general result of Sec. I for filamentary circuit elements in which it was pointed out that the negative of the rate of change of flux through the area $\mathbf{v}\times d\mathbf{s}$ swept out by a conductor length element $d\mathbf{s}$, $-\mathbf{B}\cdot(\mathbf{v}\times d\mathbf{s})$, equals the motional force on this element, $(\mathbf{v}\times\mathbf{B})\cdot d\mathbf{s}$.

There is another way to see the equivalence of $\mathcal{E}_{\text{motion}}$ and $\mathcal{E}_{\text{flux}}$ which refers back to the homopolar generator. The symmetry of electromagnetic induction demands that the emf generated by rotation of the wire structure with the magnet at rest (all relative to the inertial lab frame) must equal the emf generated by rotation of the magnet with the wire structure at rest (again, relative to the lab frame). In the latter case, it is the circuit segment consisting of the radius fixed in the magnet that rotates. Rotation of this radius is equivalent to the homopolar generator treated in Sec. II, and we know that $\mathcal{E}_{\text{motion}}$ and $\mathcal{E}_{\text{flux}}$ are equal for this case.

IV. FEYNMAN'S CHALLENGE

Figure 4 is a drawing of Feynman's original scheme and shows two extended conductors touching at a point in the

middle and connected to a galvanometer by leads connected to the left and right sides of the conductor pair. As with the Faraday disk, there is a constant magnetic field perpendicular to the system. Feynman chose a path consisting of a straight line in each conductor from the point of contact with the lead wires to the common contact point where the faces of the two extended conductors touch. A rocking motion of the conductors takes them from part (a) to part (b) of Fig. 4, where Feynman again completed the circuit through the conductors by straight lines to the common contact point which has now moved from the bottom to the top. Completing the circuit with these lines makes the change in flux and the induced emf through the circuit large and relatively independent of the curvature of the facing surfaces of the conductors. But the smaller the curvature (that is, the flatter the facing surfaces), the smaller is the rocking motion necessary to go from part (a) to part (b). So when the faces of the conductors are only slightly curved, "... the rocking can be done with small motions, so that $\mathbf{v} \times \mathbf{B}$ is very small and there is practically no emf. The 'flux rule' does not work in this case." (Ref. 14, p. 17-3).

Note that Feynman's proposed path for calculating the flux change varies continuously as the rotation takes place, but gives an answer that is clearly wrong. So a continuous change of paths is not sufficient to calculate flux change. But rather than conclude that there is something wrong with the flux rule, we should conclude that the proposed path is inappropriate for this purpose. This conclusion can be appreciated by noting that the proposed path in the left (right) conductor swings generally counterclockwise (clockwise) during the small rotation, while the actual rotational motion of the plate is clockwise (counterclockwise). In other words, despite the continuous transformation of the path, it is not in any sense anchored in the moving material. Paths that are not appropriate for $\mathbf{v} \times \mathbf{B}$ are not appropriate for a flux calculation either.

An alternative path in Fig. 4(b) would be the original segment, shown dashed, plus the curved segments on the conductor edges generated by the moving common contact point. These edge segments connect with the two original straight segments. Then the change in area enclosed after rotating would be the combined areas swept out by the two original straight segments and the curved edge segments which increase in length as the rotation takes place. The area swept out by the curved edge segments is roughly the area below the common contact point and between the facing edges, a small area that goes to zero the flatter the facing surfaces. This area is shown shaded in the bottom figure. The original straight segments sweep out similarly small areas. This alternative path is at all times fixed in the material and develops (lengthens) continuously. This idea will now be explored in detail.

Consider Fig. 5, which is a re-make of Fig. 4 but with more convenient geometry. Here, the conductor, shown shaded, is a portion of a circle of radius $\delta + \epsilon$. The position after rocking is shown in Fig. 6. In the limit $\delta \to \infty$ and angle $\alpha \to 0$ such that ϵ and $\alpha(\delta + \epsilon)$ are held constant [in Fig. 6, $\alpha(\delta + \epsilon)$ is the final height of the common contact point], the curved portions of the conductors become flatter and very little rotation is needed, corresponding to Feynman's argument. The vertices of the two conductors slide up along the lead wires as rotation takes place and push out the two leads a bit.

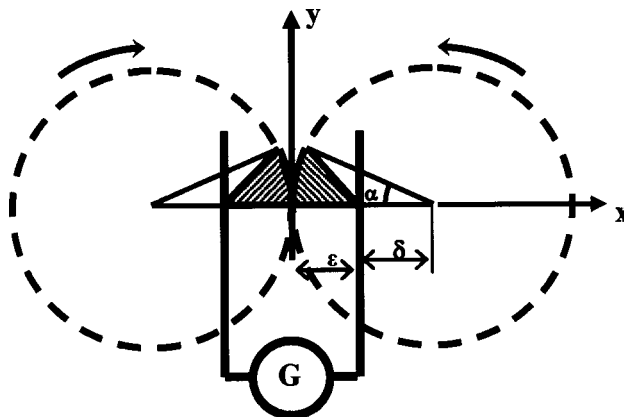Figure 7 focuses on one conductor, where the path com-

Fig. 5. A simpler geometry equivalent to that in Fig. 4. Feynman's argument for Fig. 5 correctly implies that as $\delta \to \infty$ and $\alpha \to 0$ [with ϵ and $\alpha(\delta + \epsilon)$ constant], all velocities approach zero, and the motional emf is zero. But he argued incorrectly that Faraday's law gives an incorrect finite value relatively independent of this limit.

pleting the circuit at any time t is the path at $t=0$—the segment along the lower edge of the conductor—plus the curved segment on the face that extends up to the common contact point. To repeat, all of this circuit material is fixed in the conductor, and the circuit changes (in this case lengthens) continuously throughout the rotation. Figure 7 shows the areas swept out by these segments. A calculation of the areas is tedious but not difficult. The result is

$$A = A_{P0} + A_1 + A_{BP0} - A_B, \tag{4}$$

where

$$A_{P0} = (\delta + \epsilon)^2 \left[\frac{\sin \alpha \cos \alpha}{2} - \alpha \cos \alpha + \sin \alpha - \frac{\alpha}{2} \right], \tag{5}$$

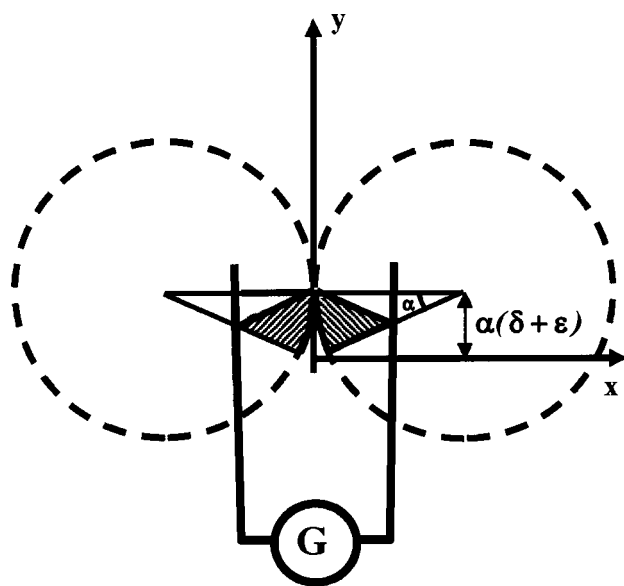$$A_1 = (\delta + \epsilon)^2 [\alpha - \sin \alpha], \tag{6}$$

Fig. 6. The geometric situation after rocking the two conductors. The two vertical lead wires to the galvanometer flare out a bit, but this flaring is common to whatever path through the conductors is chosen.
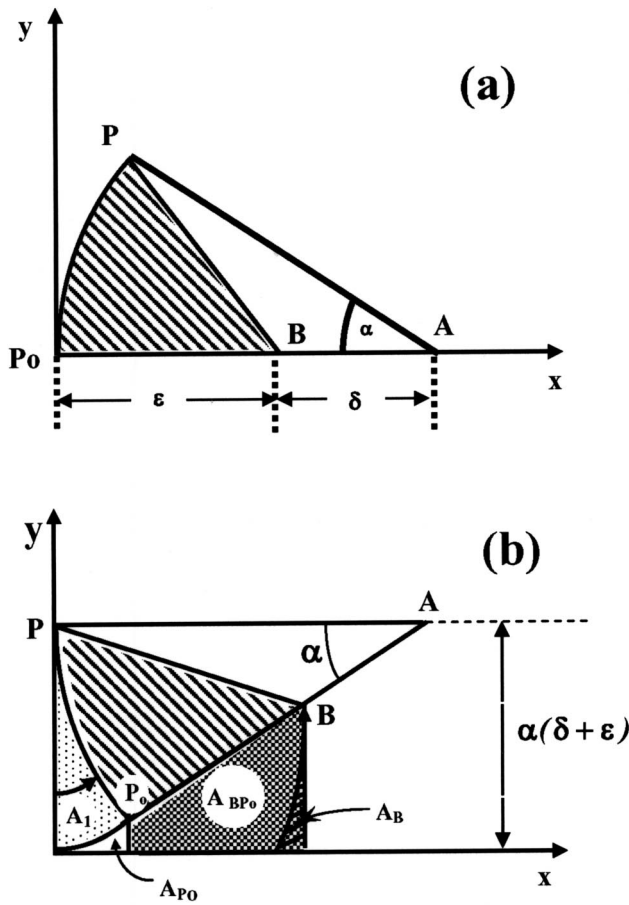
Fig. 7. (a) Initial position of one conductor. (b) As the conductor rolls up the y axis, the original circuit element, P_{0B}, is augmented by the curved segment P_{0P} to complete the circuit. These segments sweep out the relevant areas used in Faraday's law. (Note that the area A_{BP0} includes A_B.)
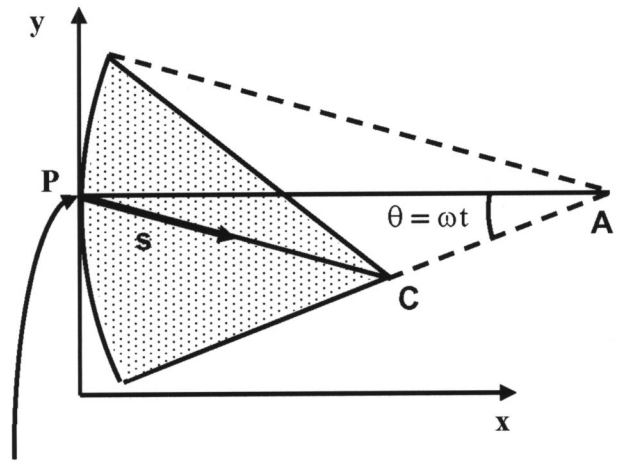
$$A_{BP0} = \frac{\epsilon \cos \alpha}{2} [2(\delta + \epsilon)\alpha - (2\delta + \epsilon)\sin \alpha], \qquad (7)$$

$$A_B = \delta(\delta + \epsilon)[\sin \alpha - \alpha \cos \alpha] - \frac{\delta^2}{2}[\alpha - \sin \alpha \cos \alpha], \qquad (8)$$

$$A = -\delta(\delta + \epsilon)\sin \alpha + \frac{\alpha}{2}[\delta^2 + (\delta + \epsilon)^2]. \qquad (9)$$

Note that the area swept out by P_0B is $A_{P0} + A_{BP0} - A_B$; A_{BP0} includes A_B. If we add the areas, multiply by 2 for the other conductor, and multiply by the magnetic field B_0, we obtain the total change in the flux through the circuit associated with the conductors. There also is a very small change in the area of the circuit due to the flaring out of the vertical leads, call it A_{leads}, but this change does not depend on the choice of paths through the conductors.

Equation (9) holds for any intermediate angle $\theta = \omega t$, which goes from 0 to $\alpha = \omega T$, where T is the time needed to do the slight rotation. If we multiply Eq. (9) by 2, substitute ωt for α and take the time derivative, we obtain the instantaneous emf, $\mathcal{E}_{\text{flux}}(t)$:

Instantaneous axis of counterclockwise rotation at P; $\mathbf{v(s)} = \boldsymbol{\omega} \times \mathbf{s}$.

Fig. 8. The force $\mathbf{v} \times \mathbf{B}$ can be calculated using the instantaneous axis of rotation and the variables from the contact point between the conductors to the point of connection to the leads. The angle θ varies from 0 to α. The length of the PC is $\sqrt{(\delta + \epsilon)^2 + \delta^2 - 2\delta(\delta + \epsilon)\cos \theta}$.

$$\mathcal{E}_{\text{flux}}(t) = -B_0 \frac{dA_{\text{leads}}}{dt} - B_0 \omega [\delta^2 + (\delta + \epsilon)^2]$$
$$+ 2B_0 \omega \delta(\delta + \epsilon)\cos \omega t. \qquad (10)$$

In the limit of $\delta \to \infty$ and $\alpha \to 0$ [with ϵ and $\alpha(\delta + \epsilon)$ constant], the angle ωt is always small, $\cos \omega t$ is always ≈ 1, A_{leads} approaches 0, and the average $\mathcal{E}_{\text{flux}}$ over the time T becomes

$$\bar{\mathcal{E}}_{\text{flux}} = -B_0 \frac{\Delta A_{\text{leads}}}{T} - B_0 \frac{\epsilon^2 \alpha}{T}. \qquad (11)$$

Thus, the effective area involved in $\bar{\mathcal{E}}_{\text{flux}}$ is roughly twice the area swept out by the length ϵ as it swings through the small angle α, which approaches 0 as $\delta \to \infty$. Thus, $\bar{\mathcal{E}}_{\text{flux}}$ is very small, which is Feynman's prediction based on $\mathbf{v} \times \mathbf{B}$.

It remains to compare Eq. (10) with the motional emf. An appropriate and convenient way to calculate $\mathcal{E}_{\text{motion}}(t)$ is to use the instantaneous axis of rotation at the contact point between the two conductors. The radius from the instantaneous axis of rotation is instantaneously fixed in the conducting material, and the transformation from one instantaneous radius to another is continuous. Therefore, all the conditions for applying Faraday's law are satisfied, even though the material used to complete the circuit is constantly changing. We can do the calculation for one of the conductors and then multiply by two. As shown in Fig. 8, the radius PC rotating about the instantaneous point of contact extends from the instantaneous axis to the point of contact of the lead. If we let s equal the length from the instantaneous axis along this radius and integrate $|\mathbf{v} \times \mathbf{B}| = \omega s B_0$ with respect to s from 0 to $\sqrt{(\delta + \epsilon)^2 + \delta^2 - 2\delta(\delta + \epsilon)\cos \theta}$ (which is the length of PC), multiply by two, and include $B_0(dA_{\text{leads}}/dt)$, we obtain the same result as in Eq. (10).

The instantaneous axis approach used for $\mathcal{E}_{\text{motion}}$ is not useful for visualizing the swept-out area, because the large upward motion of the contact point, which the instantaneous axis method ignores, obscures the total small angle through which the radius from the instantaneous axis swings (in the

same direction as the conductor swings, of course). Despite the somewhat greater complexity of the areas in Eqs. (5)–(8), the area swept out is easy to visualize and for this reason has greater pedagogical value.

V. CONCLUSIONS

Faraday's law, properly applied, can be used to calculate the induced emf in any situation where the Lorentz force can be used. It is necessary that the circuit at all times be instantaneously fixed in the conducting material and that the circuit change continuously. For extended conductors, a sufficient condition for both methods to work is that **B** be uniform and perpendicular to the chosen path, in which case any convenient path through the conductors may be chosen. There are situations where neither method works unambiguously, specifically when the emf is path-dependent.

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