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To: MSE Enthusiasts
Re: MSE Memo #83c: **Polarization Changes by flat, ideal mirrors**
Date: May 7, 2008

Introduction

We are considering a in-situ scheme to calibrate the C-MOD MSE diagnostic using polarized light sources that are translated into position after each C-MOD shot, as illustrated in Figure 1. We require two light sources (rather than just one) because the change in polarization angle due to stress-induced birefringence is a function of two variables: the actual phase shift imposed by the stress, and the orientation of the stress principal axis. By measuring the change in polarization at two different polarization angles, we can uniquely determine both the phase shift and the angle of the stress principal axis, from which one can compute the change in polarization angle at *any* input polarization angle.

The overall design objective is to provide a calibration that is accurate to better than 0.2^o in pitch angle, which requires an accuracy of better than 0.1^o in the MSE frame of reference. Unfortunately, if the calibration polarized light source were to wobble about its axis by some angle $\Delta\theta$, the polarization angle of its light changes by the same amount. This places a very demanding requirement on the mechanical design of the translatable light source: it must retain its orientation, over a period of months, to something in the neighborhood of 0.1^o.

This difficult requirement led us to consider an alternate scheme (Figure 2) that uses a *fixed* (non-moving) polarized light source that is mounted on the MSE optics cannister. The polarized light is reflected by a mirror that is translated into the MSE field-of-view after each shot. This scheme still requires that the polarized light source retain its orientation to better than 0.1^o over a period of months, but this should not be difficult to achieve because the the light source is firmly attached to the rugged MSE optics cannister.

Two questions arise: first, the light will be reflected from the mirror at non-normal incidence. How will this change the polarization angle? Second, there will be unavoidable errors in positioning the mirror. How will these errors affect the polarization angle?

The approach used here has been generalized to a train of mirrors in N. Elias, 'Optical Interferometric Polarimetry. I. Foundation', The Astrophysical Journal **549**:647-668, March 1, 2001.

The basic conclusion of this analysis is that, unfortunately, small angular displacements of the mirror generate surprisingly large changes in the polarization angle of the reflected light. For an ideal metal mirror, the mis-orientation of the mirror that we can tolerate isn't much less than the allowable mis-orientation of the polarized light source in the original calibration scheme, unless the angle-of-incidence is less than about 14^o. This result may signficantly complicate the optical design of a calibration system using a fixed polarized light source and a translated mirror.

Implementation in IDL procedure

The arithmetic described by Eqs. 1 through 14 are implemented, in the same order as the numbered equations, in the IDL procedure \ SSCOTT \ IDL \ MIRROR_REFLECTION_2007.PRO. The procedure accepts as input $\hat{k}$, $\hat{n}$, and the polarization direction ϕ_p and returns the k-vector of the reflected ray, $\hat{k}'$, and its polarization direction ϕ'_p.

The calculation

The geometry for the problem is defined in Fig. 3. Inside the tokamak, two vectors are used to define the coordinate system: the vertical unit vector $\hat{z}$ (i.e. vertical with respect to gravity) and the k-vector of the optical ray of interest, $\hat{k}$. Two other variables enter the calculation: the unit normal to the mirror surface, $\hat{n}$, and the polarization direction of the light, ϕ_p.

First, we construct a right-handed coordinate system $[\hat{x}, \hat{k}, \hat{y}]$ which is convenient for describing the optical ray's electrical field vectors. A horizontal coordinate, $\hat{x}$, is defined as the cross product of the z-axis and $\hat{k}$. The vertical coordinate, $\hat{y}$, is then defined as the cross product of the $\hat{x}$ direction and the $\hat{k}$ direction.

$$\begin{aligned} \hat{x} &= \frac{\hat{k} \times \hat{z}}{|\hat{k} \times \hat{z}|} \\ \hat{y} &= \hat{x} \times \hat{k} \end{aligned} \tag{1}$$

The electric field vector will have components only in the $\hat{x}$ and $\hat{y}$ directions. In the tokamak coordinate system, the pitch angle of the electric field vector is defined as $\phi_p \equiv \tan^{-1}(E_y/E_x)$. Note that by construction, $\hat{x}$ is horizontal in the tokamak frame of reference, but $\hat{y}$ may not be exactly vertical ($\hat{y}$ will be vertical only if $\hat{k}$ lies in the tokamak horizontal midplane). The $[\hat{x}, \hat{k}, \hat{y}]$ coordinate system is convenient for describing the optical ray's polarization and propagation as it approaches the mirror, but it is not appropriate for what happens *at* the mirror. At the mirror, we need to consider the directions of the S-polarization and P-polarization, which are defined by the the k-vector and the mirror normal, $\hat{n}$. We define a horizontal coordinate $\hat{s}$ and a vertical coordinate $\hat{p}$ corresponding to the directions of the S- and P- polarizations, respectively:

$$\begin{aligned} \hat{s} &= \frac{\hat{k} \times \hat{n}}{|\hat{k} \times \hat{n}|} \\ \hat{p} &= \hat{s} \times \hat{k} \end{aligned} \tag{2}$$

Linearly polarized light in the plasma coordinate system will have components E_x and E_y. But the reflective properties of the mirror are defined with respect to the S- and P- polarized components E_s and E_p. So we need to rotate from the x, y coordinates to the s, p coordinates.

By reference to Fig. 3, this rotation angle, β, is given by

$$\begin{aligned}
\sin\beta &= -\hat{p}\cdot\hat{x} = \hat{s}\cdot\hat{y} \\
\cos\beta &= \hat{p}\cdot\hat{y} = \hat{s}\cdot\hat{x} \\
\beta &= \tan^{-1}\left(\frac{-\hat{p}\cdot\hat{x}}{\hat{p}\cdot\hat{y}}\right) \\
&= \tan^{-1}\left(\frac{\hat{s}\cdot\hat{y}}{\hat{s}\cdot\hat{x}}\right) \\
&= \tan^{-1}\left(\frac{\hat{s}\cdot\hat{y}}{\hat{p}\cdot\hat{y}}\right) \\
&= -\tan^{-1}\left(\frac{\hat{p}\cdot\hat{x}}{\hat{s}\cdot\hat{x}}\right) \qquad (3)
\end{aligned}$$

In the original, 'optical-ray' coordinate system, the electric field vectors E_x and E_y are simply defined by the polarization angle ϕ_p:

$$\begin{aligned}
E_x &= E_o\cos\phi_p \\
E_y &= E_o\sin\phi_p \qquad (4)
\end{aligned}$$

As illustrated in Fig. 3, we can transform these coordinates into the mirror coordinate system:

$$\begin{aligned}
E_s &= E_y\sin\beta + E_x\cos\beta \\
E_p &= E_y\cos\beta - E_x\sin\beta. \qquad (5)
\end{aligned}$$

The mirror changes the electric field vectors of the optical ray in a simple way. Using primes to denote parameters of the reflected ray,

$$\begin{aligned}
E'_s &= \pm R_s E_s \qquad (6) \\
E'_p &= R_p E_p \qquad (7)
\end{aligned}$$

where R_s and R_p are the mirror's reflectivity coefficients for the S- and P-polarizations, respectively. The '+' sign pertains to dielectric mirrors and the '-' sign pertains to metal mirrors. Henceforth we will assume ideal mirror properties, i.e. $R_s = R_p = 1$.

The change in direction of the optical ray's k-vector is given by

$$\hat{k}' = \hat{k} - 2(\hat{k}\cdot\hat{n})\hat{n}. \qquad (8)$$

We now need to define the mirror coordinate system for the reflected ray. Just as we did for the incident ray, we construct a coordinate system for the reflected ray using the mirror normal $\hat{n}$ and the relevant k-vector, which in this case is the k-vector of the reflected ray. So the unit vectors for the mirror coordinate system for the reflected ray are $\hat{s}'$ and $\hat{p}'$,

$$\begin{aligned}
\hat{s}' &= \frac{\hat{k}'\times\hat{n}}{|\hat{k}'\times\hat{n}|} \\
\hat{p}' &= \hat{s}'\times\hat{k}'. \qquad (9)
\end{aligned}$$

Note that $\hat{s}' = \hat{s}$ because (focusing just on the numerator for the present):

$$\begin{aligned}
\hat{s}' &= \hat{k}' \times \hat{n} \\
&= (\hat{k} - 2(\hat{k} \cdot \hat{n})\hat{n}) \times \hat{n} \\
&= \hat{k} \times \hat{n} - 2(\hat{k} \cdot \hat{n})\hat{n} \times \hat{n} \\
&= \hat{k} \times \hat{n}
\end{aligned} \tag{10}$$

which is just the numerator for $\hat{s}$.

Similarly, we define a coordinate system $\hat{x}'$ and $\hat{y}'$ for the reflected optical ray in the lab frame using the absolute directional vector $\hat{z}$ and the k-vector of the reflected ray:

$$\begin{aligned}
\hat{x}' &= \frac{\hat{k}' \times \hat{z}}{|\hat{k}' \times \hat{z}|} \\
\hat{y}' &= \hat{x}' \times \hat{k}'
\end{aligned} \tag{11}$$

We have already computed the electric field vectors of the reflected ray in the mirror's coordinate system (E'_s and E'_p), but to compute the polarization direction in the lab frame (i.e. relative to the absolute vertical direction $\hat{z}$), we need to transform these back into the coordinate system of the optical ray. So we need to compute the rotation angle β' between these coordinate systems. Following the approach we used to evaluate the corresponding rotation angle β for the initial transformation *into* the mirror coordinate system, we evaluate β' from

$$\begin{aligned}
\beta' &= \tan^{-1}\left(\frac{-\hat{p}' \cdot \hat{x}'}{\hat{p}' \cdot \hat{y}'}\right) \\
&= \tan^{-1}\left(\frac{\hat{s}' \cdot \hat{y}'}{\hat{s}' \cdot \hat{x}'}\right) \\
&= \tan^{-1}\left(\frac{\hat{s}' \cdot \hat{y}'}{\hat{p}' \cdot \hat{y}'}\right) \\
&= -\tan^{-1}\left(\frac{\hat{p}' \cdot \hat{x}'}{\hat{s}' \cdot \hat{x}'}\right)
\end{aligned} \tag{12}$$

Finally, we transform the electric field vectors in the mirror coordinate system of the reflected ray:

$$\begin{aligned}
E'_x &= E'_s \cos\beta' - E'_p \sin\beta' \\
E'_y &= E'_s \sin\beta' + E'_p \cos\beta'
\end{aligned} \tag{13}$$

and the polarization angle of the reflected ray in the lab frame becomes

$$\phi'_p = \tan^{-1}\left(\frac{E'_y}{E'_x}\right) \tag{14}$$

We consider three configurations of the incident light & mirror orientation of increasing complexity. In the first case, the incident light is assumed to lie in the $x-y$ plane, while the mirror normal is oriented in the $\hat{x}$ direction, as illustrated in Figure 4. In the intermediate case (Figure 5), the incident light is allowed to have a non-zero component in the $\hat{z}$-direction, but we still assume the mirror is oriented with its normal in the $\hat{x}$ direction. In the final case (Figure 6), we also allow the mirror to be rotated about the $y-axis$ by an angle ϵ and then by an angle α about the z-axis.

Analytic calculation for simplified geometry: 'tilt' the mirror

We can compute the change in polarization direction analytically for a simple case where $\phi = 0, \alpha = 0$. This corresponds to a configuration where the incident light lies in the $x-y$ plane with an angle-of-incidence θ with respect to a vertical mirror. The mirror is then tilted by an angle ϵ about the y-axis. The geometry is a simplified version of the configuration illustrated in Fig. 6

For reference: this arithmetic was checked with Maple, using file reflection_simple_flat_mirror.mws.

$$
\begin{aligned}
\hat{k} &= [-\cos\theta, -\sin\theta, 0] \\
\hat{n} &= [\cos\epsilon, 0, \sin\epsilon] \\
A &= \sqrt{1-\cos^2\theta\cos^2\epsilon} \\
\hat{s} &= \frac{1}{A}[-\sin\theta\sin\epsilon, \cos\theta\sin\epsilon, \sin\theta\cos\epsilon] \\
\hat{p} &= \frac{1}{A}[\sin^2\theta\cos\epsilon, -\cos\theta\sin\theta\cos\epsilon, \sin\epsilon] \\
\hat{x} &= [-\sin\theta, \cos\theta, 0] \\
\hat{y} &= [0, 0, 1] \\
\beta &= \tan^{-1}\left(\frac{\sin\theta}{\tan\epsilon}\right) \\
\hat{k}' &= [\cos\theta\cos 2\epsilon, -\sin\theta, \cos\theta\sin 2\epsilon] \\
\hat{s}' &= \frac{1}{A}[-\sin\theta\sin\epsilon, \cos\theta\sin\epsilon, \sin\theta\cos\epsilon] \\
\hat{p}' &= \frac{1}{A}[\cos^2\theta\sin\epsilon\sin 2\epsilon + \sin^2\theta\cos\epsilon, \\
& \quad \sin\theta\cos\theta(\sin\epsilon\sin 2\epsilon + \cos^2 2\epsilon, \\
& \quad -(\sin^2\theta\sin\epsilon + \cos^2\theta\sin\epsilon\cos 2\epsilon)] \\
B &= \sqrt{1-\cos^2\theta\sin^2 2\epsilon} \\
\hat{x}' &= \frac{1}{B}[-\sin\theta, -\cos\theta\cos 2\epsilon, 0] \\
\hat{y}' &= \frac{1}{B}[-\cos^2\theta\sin 2\epsilon\cos 2\epsilon, \sin\theta\cos\theta\sin 2\epsilon, \sin^2\theta + \cos^2\theta\cos^2 2\epsilon]
\end{aligned}
$$

$$
\begin{aligned}
\beta' &= \tan^{-1}\left(\frac{\sin\theta}{\tan\epsilon(\sin^2\theta - \cos^2\theta\cos 2\epsilon)}\right) \\
E_x &= E_o\cos\phi_p \\
E_y &= E_o\sin\phi_p \\
E_p &= E_y\cos\beta - E_x\sin\beta = E_o\sin(\phi_p - \beta) \\
E_s &= E_x\cos\beta + E_y\sin\beta = E_o\cos(\phi_p - \beta)
\end{aligned} \tag{15}
$$

For an ideal dielectric mirror,

$$
\begin{aligned}
E'_p &= E_p \\
E'_s &= E_s \\
E'_x &= E'_s\cos\beta' - E'_p\sin\beta' = E_o\cos(\phi_p + \beta' - \beta) \\
E'_y &= E'_s\sin\beta' + E'_p\cos\beta' = E_o\sin(\phi_b + \beta' - \beta) \\
\phi'_p &= \tan^{-1}\left(\frac{\sin(\phi_p + \beta' - \beta)}{\cos(\phi_p + \beta' - \beta)}\right) \qquad \text{(`tilt'; dielectric)}
\end{aligned} \tag{16}
$$

Note that the angles β and β' which are used to move into and out of the mirror's S-P frame of reference enter only through their difference.

For an ideal metal mirror,

$$
\begin{aligned}
E'_s &= -E_s = -E_o\cos(\phi_p - \beta) \\
E'_p &= E_p = E_o\sin(\phi_p - \beta) \\
E'_x &= -E_o\cos(\phi_p - \beta - \beta') \\
E'_y &= E_o\sin(\phi_p - \beta - \beta') \\
\phi'_p &= \tan^{-1}\left(\frac{-\sin(\phi_p - \beta - \beta')}{\cos(\phi_p - \beta - \beta')}\right) \qquad \text{(`tilt; metal)}
\end{aligned} \tag{17}
$$

The analytic expressions in Eqs. 16 and 17 are implemented as option 'itype=1' in mirror_reflection_2007.pro.

The polarization angle of the reflected light, ϕ'_p, is given by Equation 16 for a perfect dielectric mirror:

$$
\begin{aligned}
\phi'_p &= \tan^{-1}\left(\frac{\sin(\phi_p + \beta' - \beta)}{\cos(\phi_p + \beta' - \beta)}\right) \\
&= \frac{\sin\phi_p\cos(\beta' - \beta) + \cos\phi_p\sin(\beta' - \beta)}{\cos\phi_p\cos(\beta' - \beta) - \sin\phi_p\sin(\beta' - \beta)} \\
&= \frac{\tan\phi_p + \tan(\beta' - \beta)}{1 + \tan\phi_p\tan(\beta' - \beta)}
\end{aligned} \tag{18}
$$

But we remember the angle-addition formula for the tangent,

$$
\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a\tan b} \tag{19}
$$

So Equation 18 implies that $\phi'_p = \phi_p + \beta' - \beta$, i.e. the change in polarization angle upon reflection from a dielectric mirror for the 'tilt' configuration is just $\boxed{\Delta\phi_p = \beta' - \beta}$ and this change is independent of the incident polarization angle.

For a perfect metal mirror,

$$\begin{aligned}\tan\phi'_p &= \frac{-\sin(\phi_p + (\beta - \beta'))}{\cos(\phi_p - (\beta + \beta'))} \\ &= \frac{\tan\phi_p - \tan(\beta' + \beta)}{1 + \tan\phi_p \tan(\beta' + \beta)} \qquad \text{('tilt'; metal)} \end{aligned} \tag{20}$$

and so $\phi' = \pi - (\phi - (\beta' + \beta))$, i.e. the polarization angle upon reflection from a perfect metal mirror changes by an amount $\boxed{\Delta\phi_p = \pi + \beta' + \beta - 2\phi_p}$ for the simplified 'tilt' geometry. Note that unlike the situation for a dielectric mirror, the change in polarization angle from reflection off a metal mirror *does* depend on the input polarization direction.

By reference to Figure 7, we can simplify expression for the change in polarization angle upon reflection from a mirror for the simplified 'tilt' geometry ($\phi = \alpha = 0$). First we consider the case of a dielectric mirror.

We construct two triangles with angles β' and β at the lower-left corner. The width of the triangles is $\tan\epsilon$ and the height of the triangles is $\sin\theta$ and $a\sin\theta$, where $a = 1/(\sin^2\theta - \cos^2\theta\cos 2\epsilon)$. The hypotenuses of the triangles are x and x', where

$$\begin{aligned} x^2 &= \tan^2\epsilon + \sin^2\theta \\ x'^2 &= \tan^2\epsilon + a^2\sin^2\theta \end{aligned} \tag{21}$$

From the law of sines,

$$\frac{x}{\sin(90^o - \beta')} = \frac{(a-1)\sin\theta}{\sin\Delta\beta} \tag{22}$$

and from the law of cosines,

$$(a-1)^2\sin^2\theta = x^2 + x'^2 - 2xx'\cos(\Delta\beta) \tag{23}$$

After a little arithmetic, this yields

$$\tan\Delta\phi_p = \tan\Delta\beta = \frac{(a-1)\sin\theta\tan\epsilon}{\tan^2\epsilon + a\sin^2\theta} \tag{24}$$

After some more arithmetic, we get the final expression

$$\tan\Delta\phi_p = \frac{\sin\theta\sin(2\epsilon)\cos^2\epsilon}{\tan^2\theta - \cos(2\epsilon)\sin^2\epsilon} \qquad \text{('tilt'; dielectric)} \tag{25}$$

The expression for $\Delta\phi_p$ given by Eq. 25 is implemented as 'itype=2' in mirror_reflection_2007.pro and it agrees with the numerically-evaluated result. Note that we should expect to see large changes in the polarization angle when the denominator of Eq. 25 is small, i.e. when $\tan^2\theta \approx \cos(2\epsilon)\sin^2\epsilon$.

In the limit where both θ and ϵ are small, Eq. 25 reduces to

$$\tan\Delta\phi_p \approx \frac{2\theta\epsilon}{\theta^2-\epsilon^2} \qquad \text{(`tilt'; dielectric)} \tag{26}$$

This is implemented as 'itype=3' in mirror_reflection_2007.pro. Note that in this limit, we expect to see large changes in the polarization angle when $\theta \approx \epsilon$.

In many of our numerical calculations, we have investigated configurations with a 'modest' incidence angle ($\theta \approx 15^o$) and 'small' tilt angles ($\epsilon \approx 4-8^o$). If we take the limit of Eq. 26 with $\theta >> \epsilon$ (but both angles still small), we get

$$\tan\Delta\phi_p \approx \frac{2\epsilon}{\theta}. \qquad \text{(`tilt'; dielectric)} \tag{27}$$

Equation 27 implies that the change in polarization angle in this limit (a) increases linearly with the tilt angle ϵ; and (b) increases *inversely* with the angle-of-incidence θ. This result agrees qualitatively with our numerical calculations.

Now, for a metal mirror, we recall that the change in polarization angle is $\Delta\phi_p = \pi + (\beta'+\beta) - 2\phi_p$. Using the angle-addition formula for the tangent, we get:

$$\begin{aligned}
\tan(\beta'+\beta) &= \frac{\tan\beta' + \tan\beta}{1-\tan\beta'\tan\beta} \\
&= \frac{\frac{\sin\theta}{\tan\epsilon(\sin^2\theta-\cos^2\theta\cos(2\epsilon))} + \frac{\sin\theta}{\tan\epsilon}}{1-\frac{\sin^2\theta}{\tan^2\epsilon(\sin^2\theta-\cos^2\theta\cos(2\epsilon)))}} \\
&= \frac{\sin\theta\tan\epsilon(1+\sin^2\theta-\cos^2\theta\cos(2\epsilon)))}{\tan^2\epsilon(\sin^2\theta-\cos^2\theta\cos(2\epsilon))-\sin^2\theta}
\end{aligned} \tag{28}$$

so the change in polarization angle is

$$\Delta\phi_p = \pi - 2\phi_p + \tan^{-1}\left(\frac{\sin\theta\tan\epsilon(1+\sin^2\theta-\cos^2\theta\cos(2\epsilon)))}{\tan^2\epsilon(\sin^2\theta-\cos^2\theta\cos(2\epsilon))-\sin^2\theta}\right) \quad \text{(`tilt'; metal)} \tag{29}$$

This is implemented as 'itype=2' in mirror_reflection_2007.pro. In the limit of small θ and small ϵ, Eq. 29 reduces to

$$\tan\Delta\phi_p = -2\theta\epsilon \qquad \text{(`tilt'; metal)} \tag{30}$$

which is implemented as 'itype=3'.

Analytic calculation for simplified geometry: 'wobble' the mirror

Now we consider an alternate simplified configuration. As before, we assume that the incident light has no component in the vertical direction, i.e. the k vector lies in the $x-y$

plane of Figure 6. But for this case we assume that the mirror is rotated only about the z axis by an angle α, with no rotation about the y axis. This situation corresponds to $\phi = 0, \epsilon{=}0$ and is designated the 'wobble' configuration.

Below, we work though the formal arithmetic, but the configuration is really very simple: changing the 'wobble' angle α is equivalent to changing the angle-of-incidence θ while leaving the mirror pointed in the x-direction. The result for this case is well known: there is no change of polariztion direction for a dielectric mirror, while a metal mirror simply reverses the sign of the incident polarization angle.

$$
\begin{aligned}
\hat{k} &= [-\cos\theta, -\sin\theta, 0] \\
\hat{n} &= [\cos\alpha, \sin\alpha, 0] \\
m &\equiv 1 \quad \text{for} \quad \theta > \alpha \qquad \text{and} \qquad = -1 \quad \text{for} \quad \theta < \alpha \\
\hat{s} &= [0, 0, m] \\
\hat{p} &= [\sin\theta\sin(\theta-\alpha), -\cos\theta\sin(\theta-\alpha), 0]/|\sin(\theta-\alpha)| \\
\hat{x} &= [-\sin\theta, \cos\theta, 0] \\
\hat{y} &= [0, 0, 1] \\
\beta &= \pi/2 \quad \text{for} \quad \theta > \alpha \qquad \text{and} \qquad = -\pi/2 \quad \text{for} \theta < \alpha \\
\hat{k}' &= [2\cos\alpha\cos(\theta-\alpha) - \cos\theta, 2\sin\alpha\cos(\theta-\alpha) - \sin\theta, 0] \\
\hat{s}' &= [0, 0, m] \\
\hat{p}' &= [-m(2\sin\alpha\cos(\theta-\alpha) - \sin\theta), m(2\cos\alpha\cos(\theta-\alpha) - \cos\theta), 0] \\
\beta' &= \pi/2 \quad \text{for} \quad \theta > \alpha \qquad \text{and} \qquad = -\pi/2 \quad \text{for} \theta < \alpha \\
E_x &= E_o\cos\phi_p \\
E_y &= E_o\sin\phi_p \\
E_p &= -mE_o\cos\phi_p \\
E_s &= mE_o\sin\phi_p \\
E'_p &= -mE_o\cos\phi_p \\
E'_s &= mnE_o\sin\phi_p \\
E'_x &= E_o\cos\phi_p \\
E'_y &= nE_o\sin\phi_p
\end{aligned}
\tag{31}
$$

where $n = 1$ for a dielectric mirror and $n = -1$ for a metal mirror. The polarization angle after reflection is then

$$
\begin{aligned}
\phi'_p &= \phi_p \qquad \text{('wobble'; dielectric)} \\
\phi'_p &= -\phi_p \qquad \text{('wobble'; metal)}
\end{aligned}
\tag{32}
$$

i.e. there is no change in polarization direction upon reflection from a dielectric mirror, and the polarization direction simply reverses sign upon reflection from a metal mirror.

Angle measured by MSE

The calculation above is correct, so far as it goes: it computes the polarization angle of the light that is reflected from a mirror in an absolute coordinate frame: the reference frame of the torus. But there is one small detail that needs to be addressed: in our gedaken experiment that considers the effect of an angular displacement of the calibration mirror, the remainder of the MSE optical system remains fixed, and so the MSE optical axis remains fixed. If the calibration mirror's orientation changes, then the light that is reflected back into the MSE optics will no longer be coincident with the MSE optical axis. Instead, the light will enter the MSE polarimeter at some non-zero angle-of-incidence. Does the angle-of-incidence affect the angle that MSE actually MEASURES, and so is it possible that this effect magically cancels the change in polarization angle at the mirror?

To address this question, one can consider an 'ideal polarimeter' to be nothing more than a rotatable, perfect linear polarizer followed by an ideal photon counter. In this idealized system, one can determine the polarization angle of the incident light by identifying the angle at which the measured photon count rate is maximized.

So the effect of angle-of-incidence on an ideal polarimeter is just the same as the effect of angle-of-incidence on an ideal linear polarizer. We measured this effect experimentally some time ago, and we found that the change in polarization angle varies *quadratically* with the angle-of-incidence for light incident on a linear polarizer. Therefore, the polarization angle that a polarimeter measures at non-normal incidence differs only negligibly from the true polarization angle if the angle-of-incidence is small. In the case of interest here, the angle-of-incidence caused by a mirror mis-alignment would be only a few tenths of a degree.

So the change in polarization angle by reflection from a mirror will *not* be magically cancelled by the effect of non-zero angle-of-incidence of light entering the MSE optics.

Implications for Design of In-Situ Calibration System

Figure 8 is an idealized schematic of the proposed in-vessel calibration system, showing two possible locations (points **P** and **Q**) of a polarized light source. In the figure, point **P** lies in the $x - y$ plane, as does the center of lens L1. Point **Q** lies off the midplane. Light from point **P** has $\phi = 0$ while light from point **Q** has $\phi \neq 0$.

Two angular errors of orienting the mirror are considered: 'tilt' and 'wobble'. Our analysis above has shown that the change in polarization angle for a 'wobble' does not depend on the angle α (for both perfect dielectic and metal mirrors), and so *errors in positioning the mirror angle α will not cause the polarization angle to change.* In other words, the calibration system is insensitive to errors in the α angle (at least for $\phi = 0$).

The major problem for the calibration system lies in 'tilting' mis-orientations of the mirror, i.e. when $\epsilon \neq 0$. The relevant parameter is the rate at which the change in polarization angle changes with ϵ. This quantity, $d(\Delta\phi_p)/d\epsilon$, is effectively an error multiplication factor: if say $d(\Delta\phi_p)/d\epsilon = 2$, then every 0.1^o mis-orientation of the mirror in the ϵ direction will cause a 0.2^o change in polarization angle.

The original proposal for an in-situ calibration system envisioned a polarized light source (e.g. a wire grid polarizer illuminated from behind with fiber optics) that would be slid in front of the MSE L1 lens after a shot. This system obviously has an error multiplication factor of 1.0 for angular rotations about the x-axis, which then requires that the mechanical design be capable of reproducibly orienting the polarizer to an accuracy of 0.1^o if we want a calibration accuracy of 0.1^o. This requirement is very stringent and difficult to achieve given the environment: vacuum, temperature variations, disruptions, etc., and this fact led us to consider the alternate scheme in which a *fixed* polarized light source is mounted on the MSE optics cannister.

But unless a mirror has an error multiplication factor substantially less than unity, it really doesn't significantly reduce the difficulty of mechanical design. A mirror system is affected by *different* angular mis-orientations than a wire-grid polarizer: the mirror is affected by rotations about the y-axis, while the wire-grid polarizer would be affected by rotations about the x-axis. Unless it is somehow easier to eliminate mis-alignments in one direction compared to another, the real driver for the design is the error multiplication factor.

Dielectric mirror: Figure 9 plots the change in polarization angle as a function of ϵ and angle-of-incidence θ. Figure 10 plots the corresponding error multiplication factor $d(\Delta\phi_p)/\epsilon$. Note that $d(\Delta\phi_p)/\epsilon$ is large for small angles of incidence, i.e. it would greatly magnify any 'tilt' mis-orientations of the mirror. We will define a minimum performance target for a fixed-polarizer calibration system that $d(\Delta\phi_p)/\epsilon < 0.5$.

This target is met for dielectric mirrors only for $\boxed{\theta > 62^o}$, i.e. very large angles of incidence.

Metal mirror: Figure 11 and 12 plot the same quantities for a perfect metal mirror. Here, the situation is reversed: the error multiplication factor is zero at normal incidence ($\theta = 0$) and increases with increasing θ.

For metal mirrors, the target performance $d(\Delta\phi_p)/\epsilon < 0.5$ is achieved only for $\boxed{\theta < 14^o}$.

A related issue is whether the behavior changes qualitatively if the incident light has some vertical component, i.e. $\phi \neq 0$. Figures 13 and 14 plot the change in polarization angle and error multiplication factor for a single angle-of-incidence ($\theta = 10^o$) as a function of the tilt angle ϵ and ϕ. Evidently, allowing the incident light to have a k-vector with a non-zero component in the vertical direction does not change the error multiplication factor very much.

Conclusion: An in-situ calibration system that uses a fixed polarization light source mounted on the MSE optics cannister which reflects light off a sliding mirror will not be significantly easier to construct – to the desired calibration reproducibility - than a 'straightforward' system that simply positions a wire grid polarizer directly in front of the L1 lens unless the angle of incidence of light is (a) greater than about 60^o for a dielectric mirror; or (b) less than 14^o for a metal mirror. This adds a significant constraint to the design of the sliding-mirror calibration system, especially considering the fact that MSE's field-of-view spans about $30 - 35^o$.

Analytic calculation of beta

The IDL procedure MIRROR_REFLECTION_2007.PRO computes the rotation angle β numerically from various dot products, but for comparison we can also evaluate it analytically.

$$\begin{aligned}
\cos\beta &= \hat{p}\cdot\hat{y} && (33)\\
&= \left(\frac{(\hat{k}\times\hat{z})\times\hat{k}}{|\hat{k}\times\hat{z}|}\right)\cdot\left(\frac{(\hat{k}\times\hat{n})\times\hat{k}}{|\hat{k}\times\hat{n}|}\right)\\
&= \frac{\left[(\hat{k}\times\hat{n})\times\hat{k}\right]\cdot\left[(\hat{k}\times\hat{z})\times\hat{k}\right]}{|\hat{k}\times\hat{z}|\;|\hat{k}\times\hat{n}|}\\
&= \frac{\left[\hat{k}(\hat{k}\cdot\hat{n})-\hat{n}\right]\cdot\left[(\hat{k}(\hat{k}\cdot\hat{z})-\hat{z}\right]}{|\hat{k}\times\hat{z}|\;|\hat{k}\times\hat{n}|}\\
&= \frac{\hat{n}\cdot\hat{z}-(\hat{k}\cdot\hat{z})(\hat{k}\cdot\hat{n})}{|\hat{k}\times\hat{z}|\;|\hat{k}\times\hat{n}|} && (34)
\end{aligned}$$

Similarly, we can evaluate $\sin\beta$:

$$\begin{aligned}
\sin\beta &= -\hat{p}\cdot\hat{x} && (35)\\
&= -\left(\frac{(\hat{k}\times\hat{n})\times\hat{k}}{|\hat{k}\times\hat{n}|}\right)\cdot\left(\frac{\hat{k}\times\hat{z}}{|\hat{k}\times\hat{n}|}\right)\\
&= -\frac{\left[\hat{n}-\hat{k}(\hat{k}\cdot\hat{n})\right]\cdot\left[\hat{k}\times\hat{z}\right]}{|\hat{k}\times\hat{z}|\;|\hat{k}\times\hat{n}|}\\
&= \frac{-\hat{n}\cdot(\hat{k}\times\hat{z})}{|\hat{k}\times\hat{z}|\;|\hat{k}\times\hat{n}|} && (36)
\end{aligned}$$

So

$$\tan\beta = \frac{-\hat{n}\cdot(\hat{k}\times\hat{z})}{\hat{n}\cdot\hat{z}-(\hat{k}\cdot\hat{z})(\hat{k}\cdot\hat{n})} \qquad (37)$$

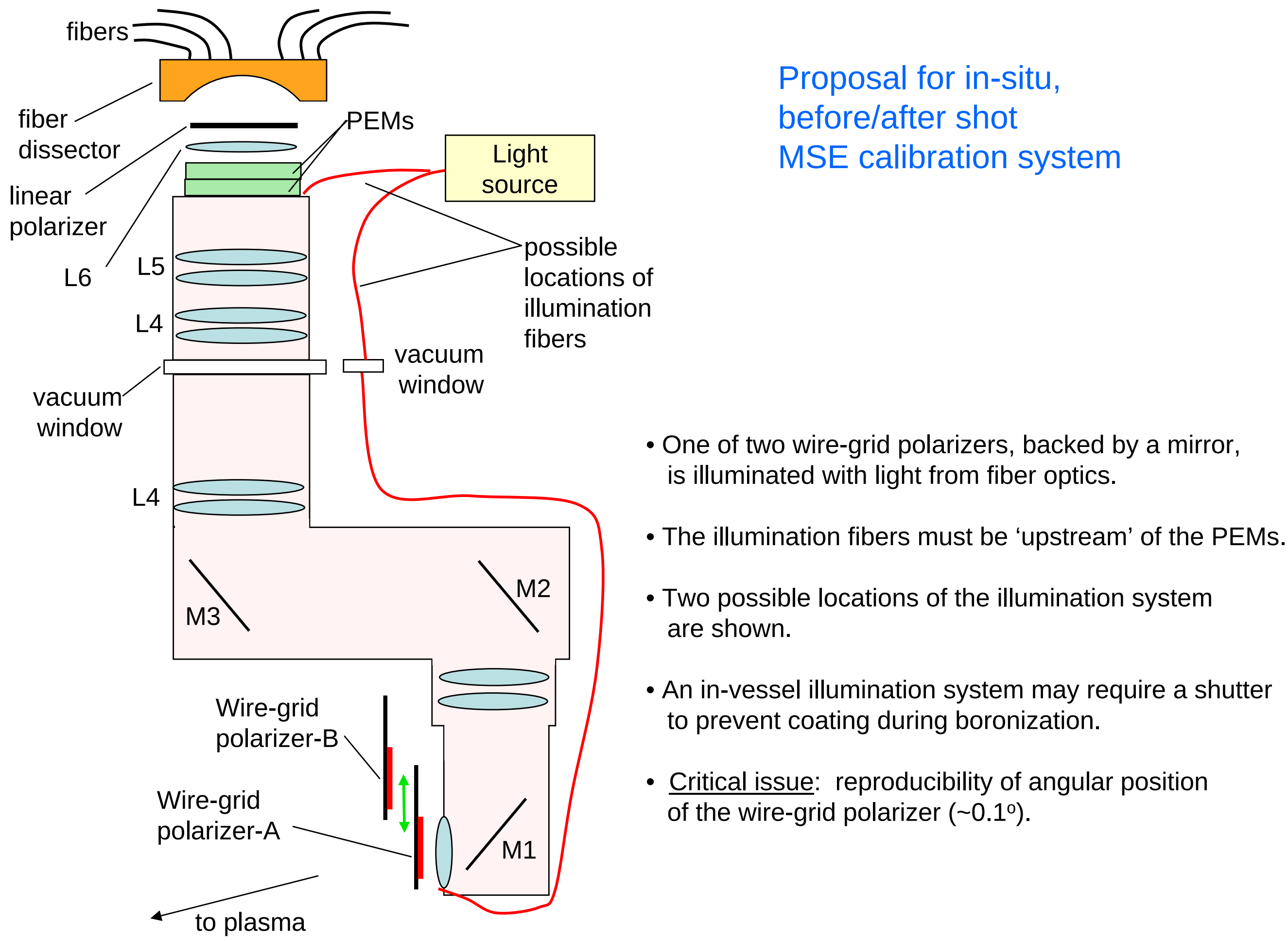

Figure 1: Proposed system for an in-situ calibration system for MSE on Alcator sc c-mod that uses a polarized light source that can be translated into the MSE field-of-view after each C-MOD plasma shot.

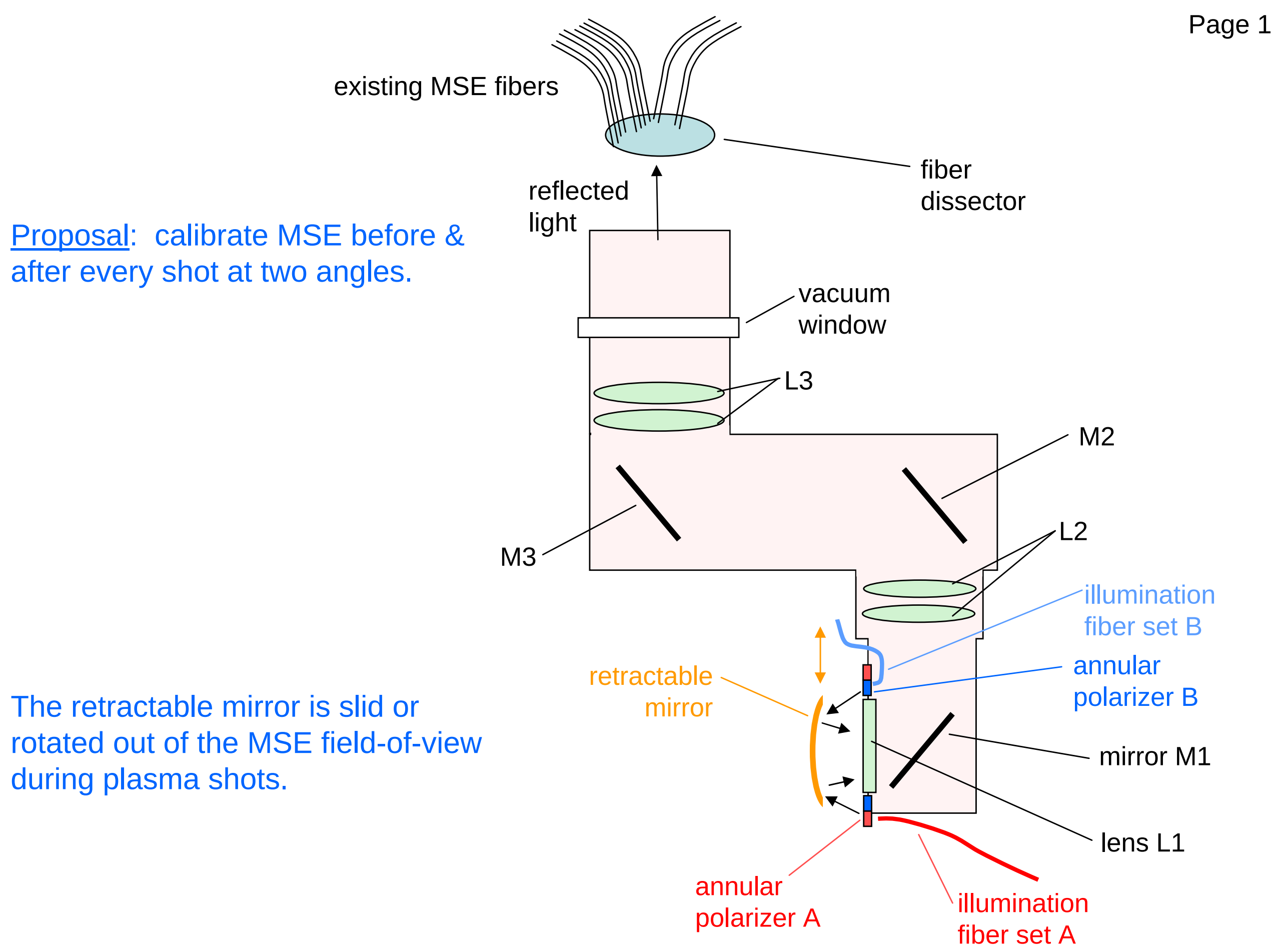

Figure 2: Proposed system for an in-situ calibration system for MSE on Alcator sc c-mod. Linearly polarized light strikes a mirror that is slid in front of the plasma-facing lens shortly after a shot.

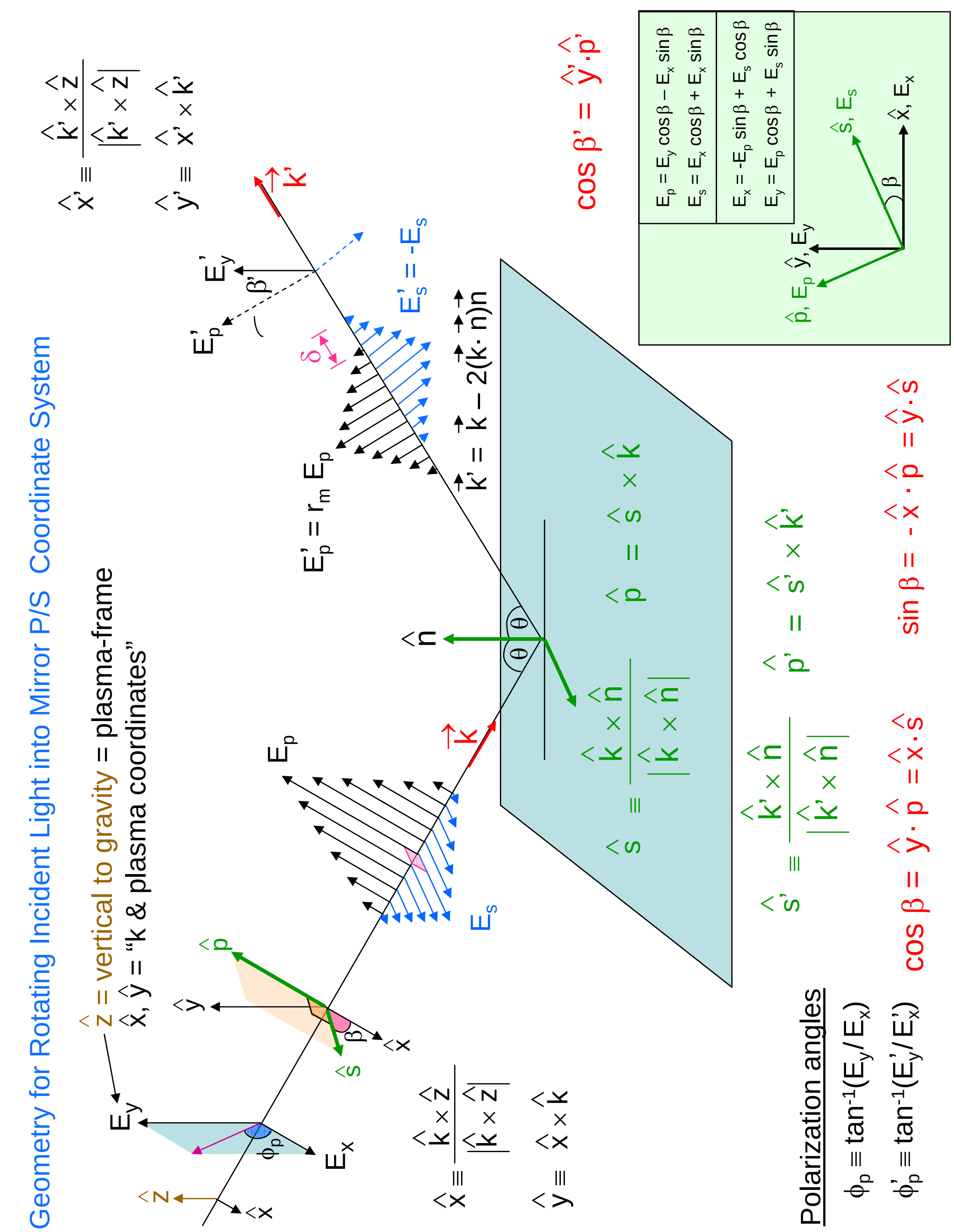

Figure 3: Geometry for the calculation.

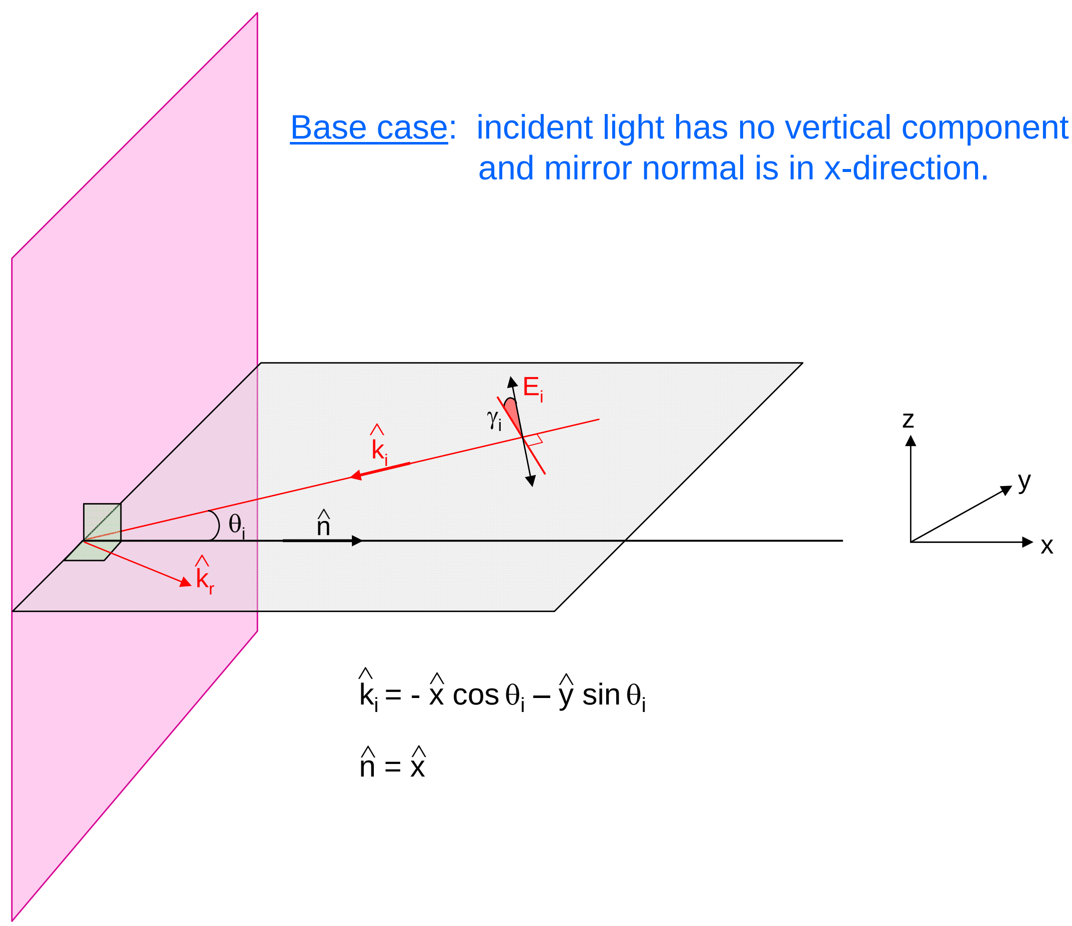

Figure 4: Geometry for computing the polarization upon reflection from an ideal, flat mirror. For this case, the k-vector of the incident light is assumed to lie in the $x - y$ plane, and the mirror normal is assumed to be in the $\hat{x}$ direction.

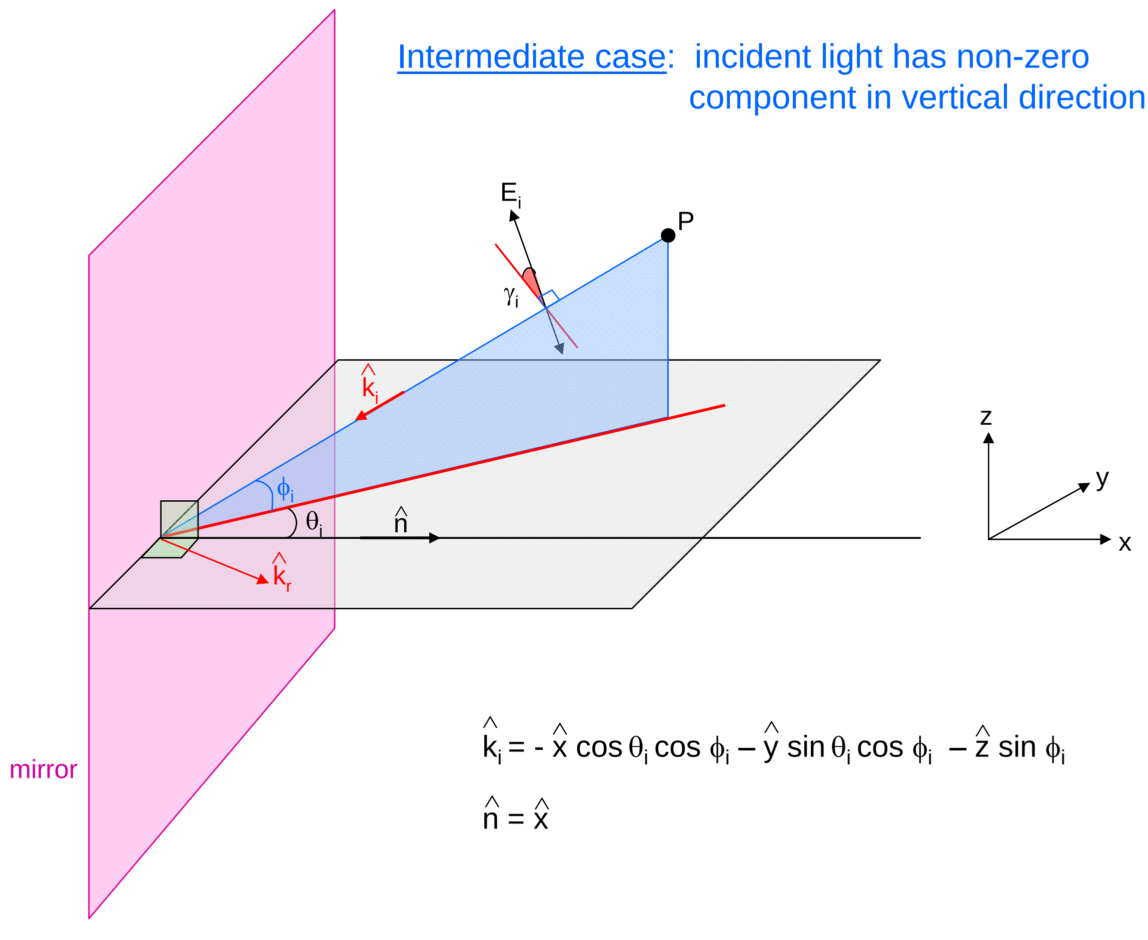

Figure 5: Geometry for computing the polarization upon reflection from an ideal, flat mirror. For this case, the k-vector of the incident light is allowed to have a non-zero component in the z - direction, but the mirror normal is still assumed to be in the $\hat{x}$ direction.

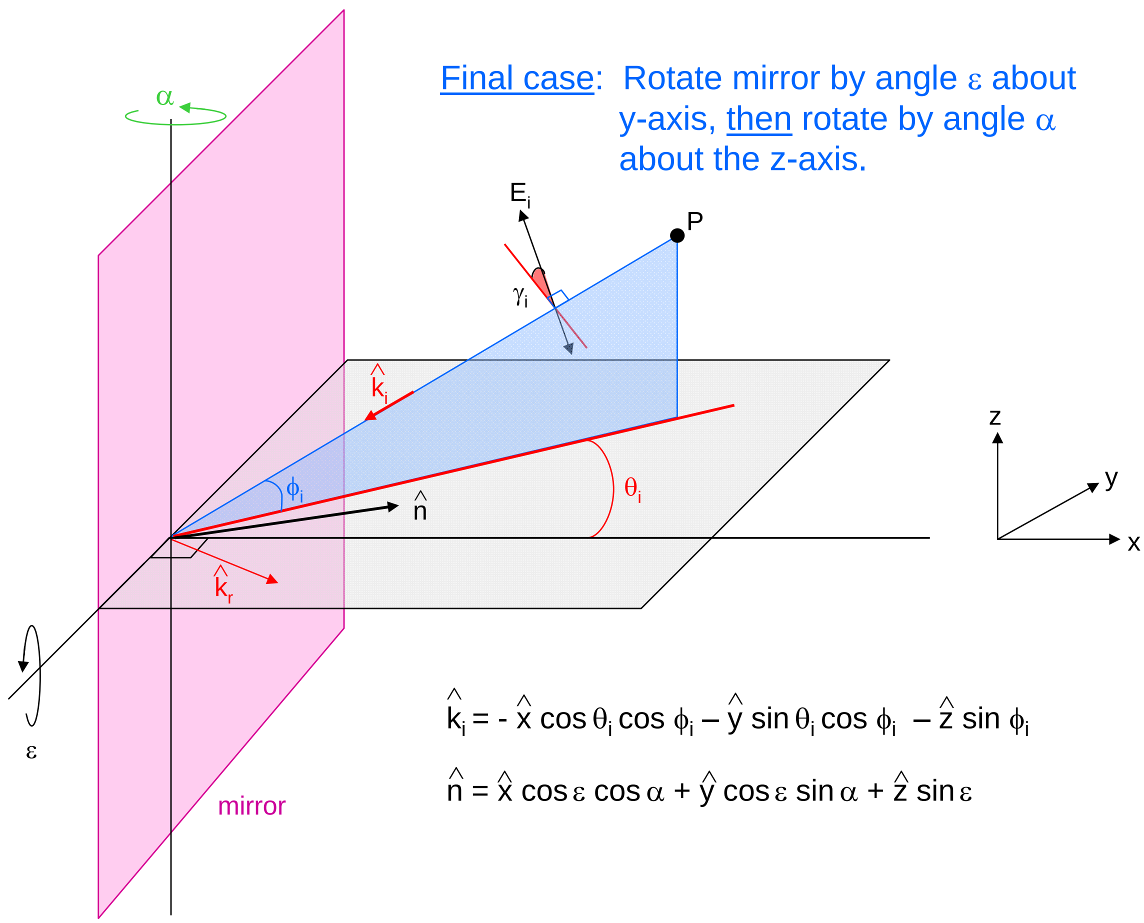

Figure 6: Geometry for computing the polarization upon reflection from an ideal, flat mirror. For this case, the k-vector of the incident light is allowed to have a non-zero component in the z - direction. The mirror is rotated an angle ϵ about the y-axis and then the mirror is rotated by an angle α about the z-axis.

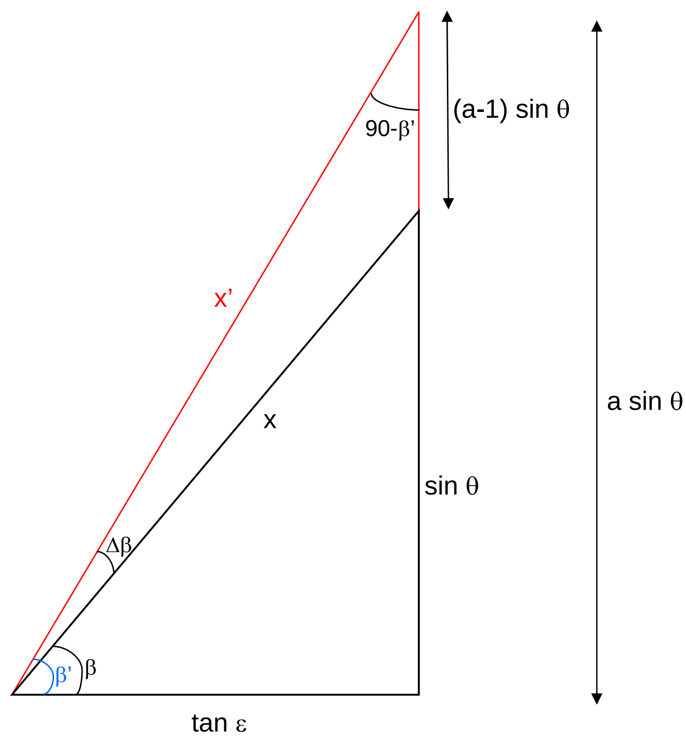

Figure 7: Geometry for computing the change in polarization angle.

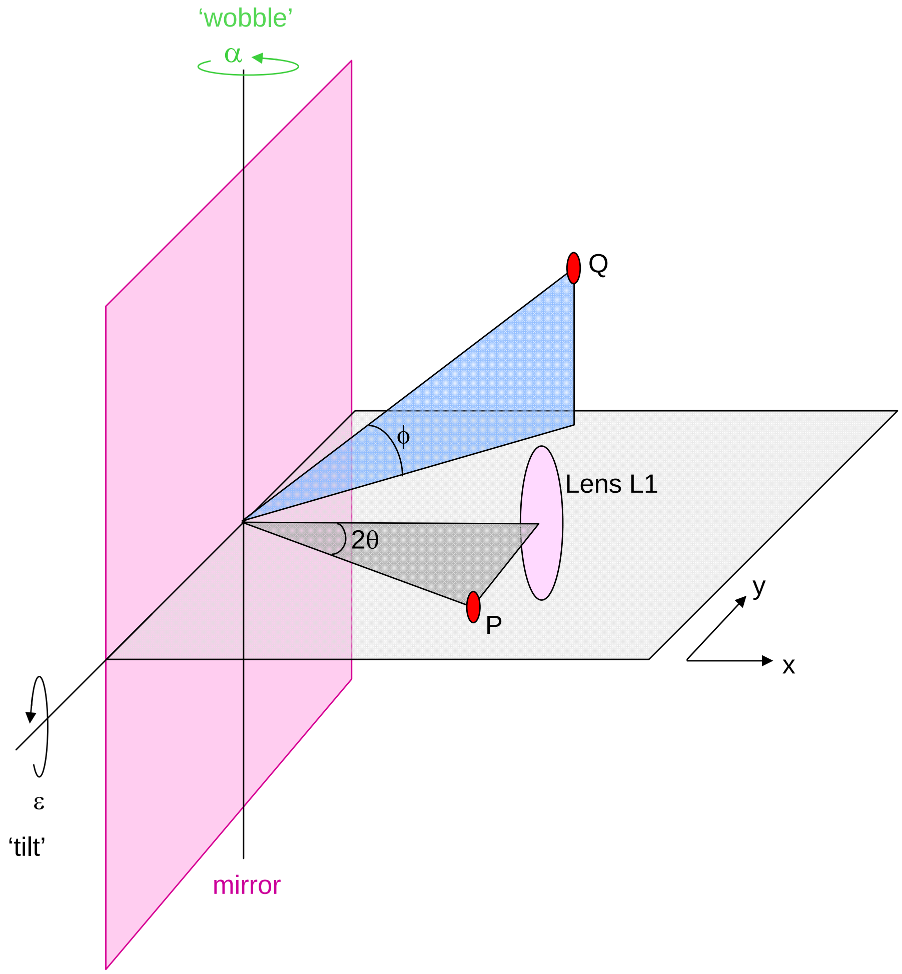

Figure 8: Geometry for illuminating the MSE plasma-facing lens.

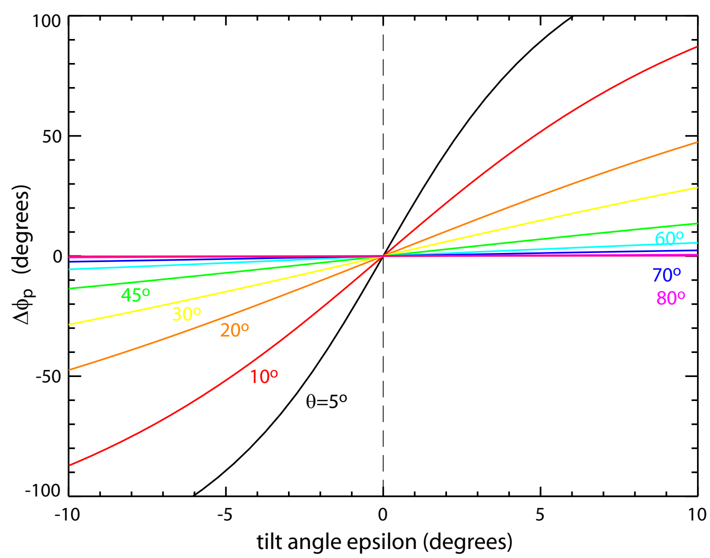

Figure 9: Change in polarization angle upon reflection from a perfect dielectric mirror for the simplified 'tilt' geometry ($\phi = \alpha = 0$) as a function of the tilt angle ϵ and the angle-of-incidence θ.

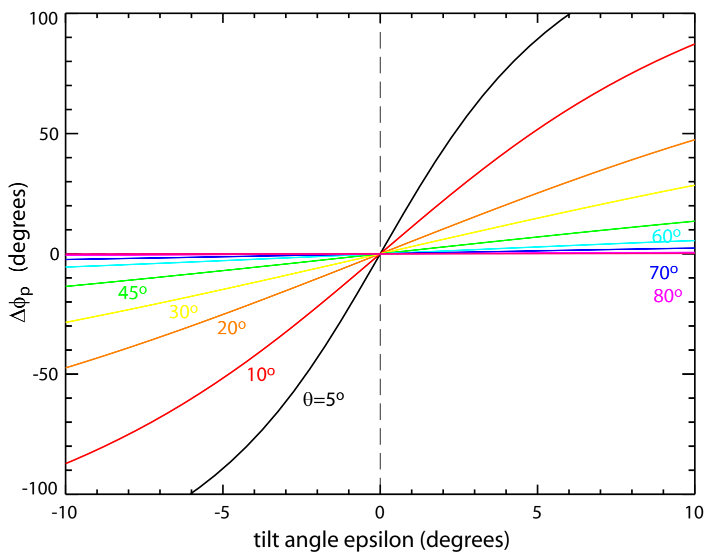

Figure 10: Absolute value of the rate-of-change of polarization angle as tilt angle is varied, $dD\phi_p/d\epsilon$ upon reflection from a perfect dielectric mirror for the simplified 'tilt' geometry ($\phi = \alpha = 0$) as a function of the tilt angle ϵ and the angle-of-incidence θ.

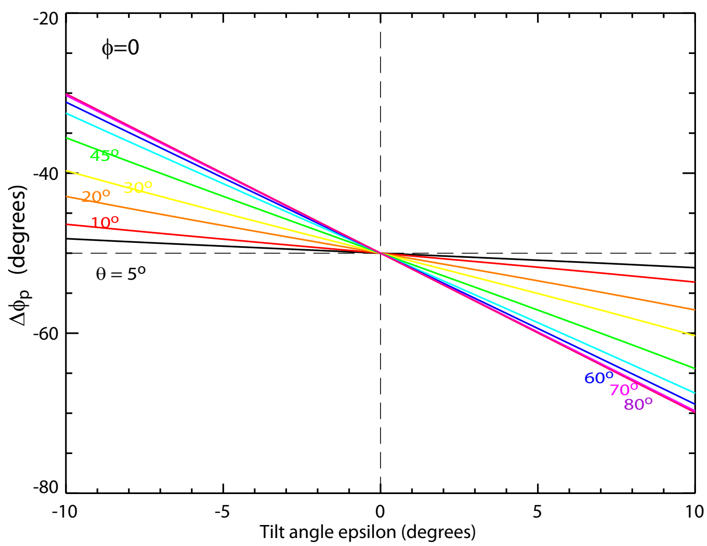

Figure 11: Change in polarization angle upon reflection from a perfect metal mirror for the simplified 'tilt' geometry ($\phi = \alpha = 0$) as a function of the tilt angle ϵ and the angle-of-incidence θ. For these calculations the input polarization angle $\phi_p = 25^o$.

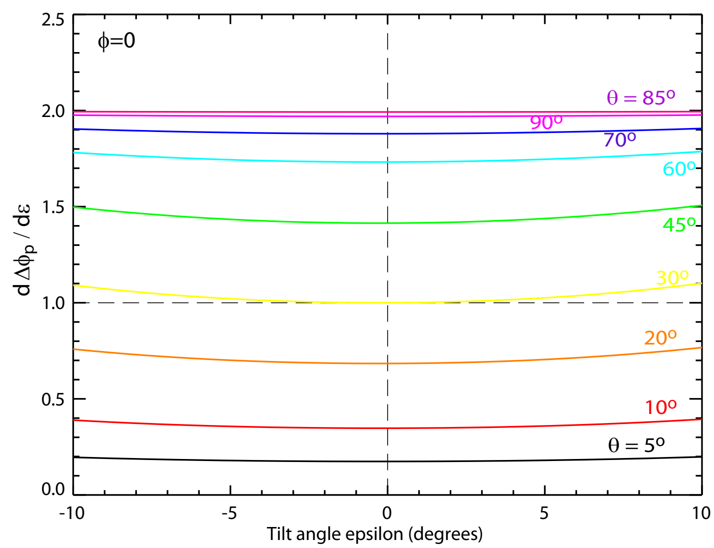

Figure 12: Absolute value of the rate-of-change of polarization angle as tilt angle is varied, $dD\phi_p/d\epsilon$ upon reflection from a perfect metal mirror for the simplified 'tilt' geometry ($\phi = \alpha = 0$) as a function of the tilt angle ϵ and the angle-of-incidence θ.

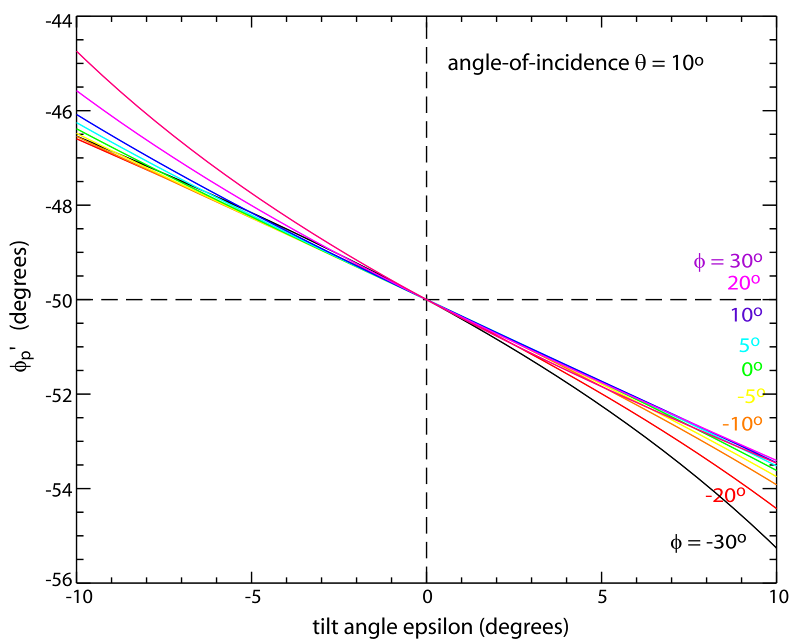

Figure 13: Change in polarization angle upon reflection from a perfect metal mirror for an angle-of-incidence $\theta = 10^o$ as a function of the 'elevation angle' ϕ and the tilt angle ϵ. For these calculations the input polarization angle $\phi_p = 25^o$.

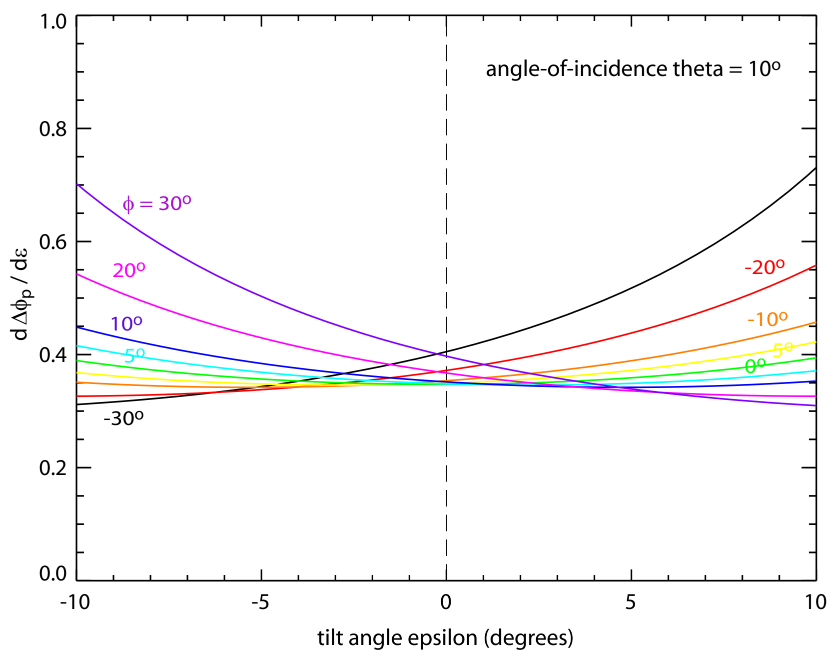

Figure 14: Absolute value of the rate-of-change in polarization angle as the tilt angle is varied, $dD\phi_p/d\epsilon$ upon reflection from a perfect metal mirror for an angle-of-incidence $\theta = 10^o$ as a function of the 'elevation angle' ϕ and the tilt angle ϵ. For these calculations the input polarization angle $\phi_p = 25^o$.