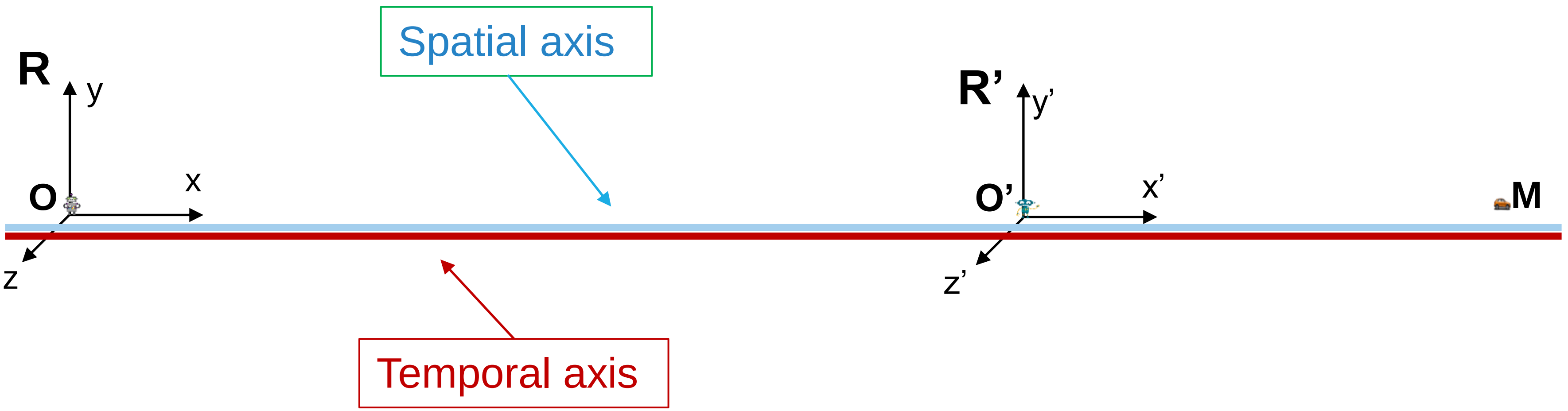
The Galilei transformations describe how the coordinates of a point M change when moving from a R referential considered at relative rest to a R' referentaial in uniform rectilinear motion with speed v. În the case of a point located on the abscissas axis of the referentials R, R', Galilean transformations are expressed by the relations:

$$x' = x - v\,t, \quad y' = y, \quad z' = z$$

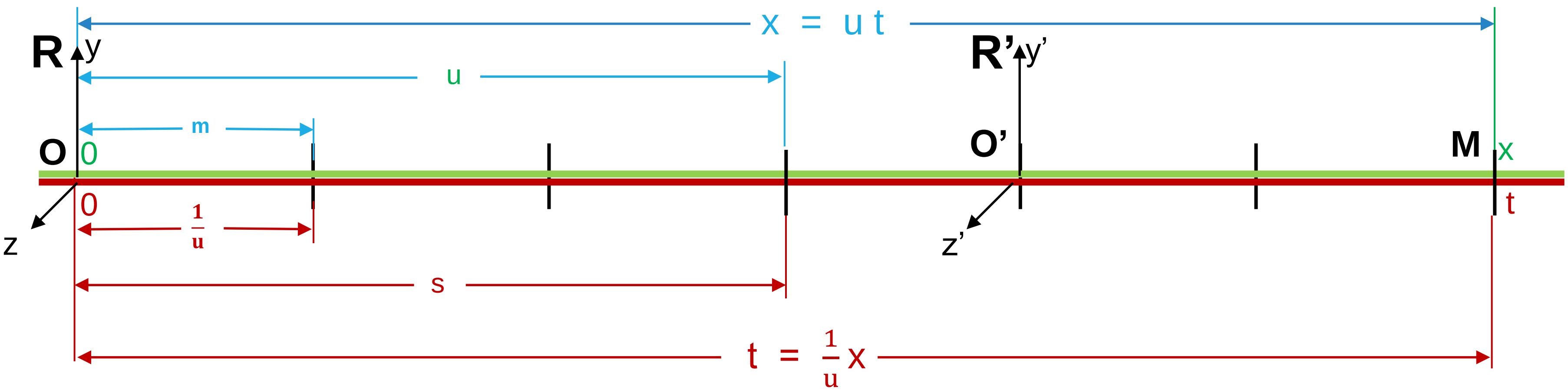
where x, y, z are the coordinates of the point M in the referential R, and x', y', z' are the coordinates of the point M in the referential R'. To these relations, classical mechanics also added the equality t' = t, with the meaning that time is not affected by the change of the reference system. This is actually a consequence of the assumption of instantaneous propagation with infinite speed of interactions, which is a fundamental assumption of classical mechanics. The existence of an infinite speed of propagation of interactions allows the synchronization of the clocks in the referentials R, R' and therefore justifies the equality t' = t.



But apart from time related to clocks, there is also a time related to the distances traveled by a point in a reference system. Distances covered in various time intervals can be highlighted on a spatial axis, that is, on a line on which a unit of length (a line segment assigned this role) and a Cartesian coordinate system are defined. Similarly, the time intervals in which different distances are traveled can be highlighted on a temporal axis, i.e. on a line on which a time unit (a line segment assigned this role) and a Cartesian coordinates system are defined.

As a concrete example, we will refer to a mobile M and to a platform R' moving in the same sense on a straight road R, the origins O, O' of the referentials R, R' being two landmarks fixed on the road and respectively on the platform. The coordinates near the mobile M are generically denoted by x, t, where x is the number of units of length (m) between the landmark O and the mobile M on the spatial axis, and t is the number of units of time (s) between the landmark O and the mobile M on the temporal axis. Also, denoting by u the distance traveled by the mobile M in the unit of time and by 1/u the time in which the mobile M travels a unit of length, we find that there are t distances of size u between the landmark O and the mobile M on the spatial axis (the mobile M travels distance x with speed u in time t on the road) and respectively x time intervals of size 1/u on the temporal axis. Therefore, the relations result

$$x = u\,t, \quad t = \frac{1}{u}x \qquad (1)$$



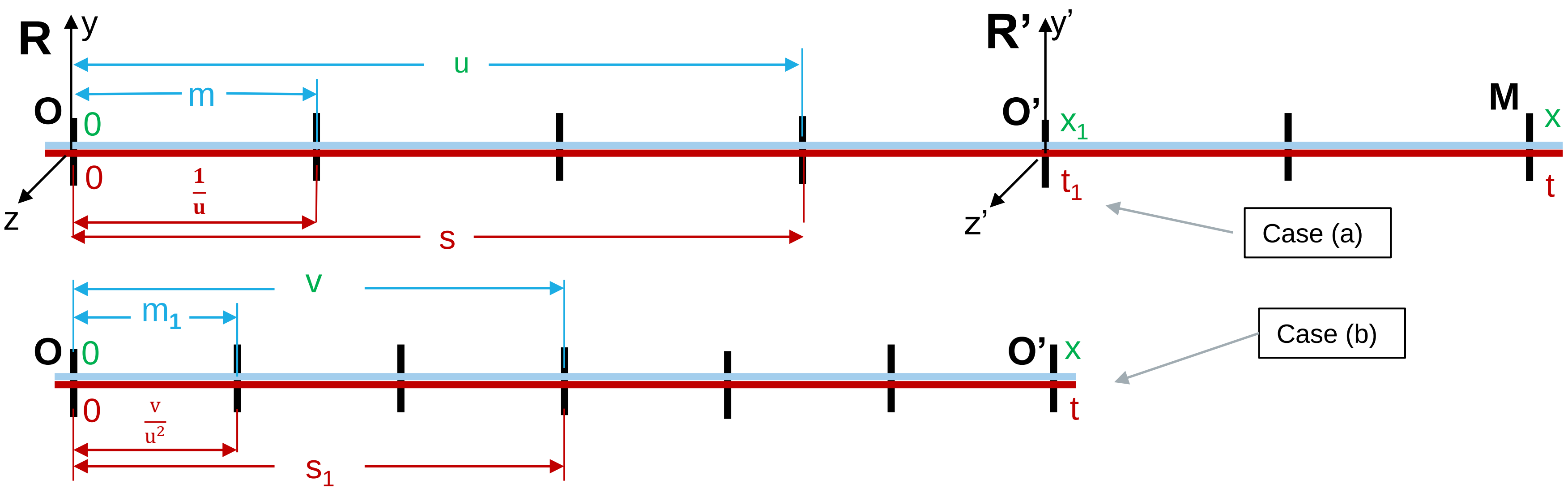
We consider the axis of the abscissas of the referential R both as a spatial axis on which we highlight the distances traveled by the mobile M in various time intervals, such as the time unit s, time 1/u or time t, and as a temporal axis on which we highlight the time intervals in which mobile M travels various distances, such as unit of length m, distance u, or distance x.

The movement of the landmark O' in relation to the landmark O can be identified with a contraction of the movement of the mobile M in relation to the landmark O. For example, if we imagine that the movements of the landmark O' and the mobile M in relation to O are played on the screens of some monitors of different sizes, we will find that the movement of the landmark O' on the larger screen is identified with the movement of the mobile M on the smaller screen. In this case, we perceive the visual difference between the movements of the landmark O' and the mobile M in two ways, as a change (contraction) of the coordinates, in which case the landmark O' is associated with the coordinates x_1, t_1 expressed by the relations:

(a) $$x_1 = \alpha x, \quad t_1 = \alpha t$$

where α is a subunit positive number, or as a modification (contraction) of the units of measure, in which case the landmark O' traverses the units of measure m_1, s_1 expressed by the relations:

(b) $$m_1 = \alpha m, \quad s_1 = \alpha s$$



In case (b), not only the measurement units are contracted with the α factor, but also any distance or time interval. Thus, the distance u becomes

$$\alpha u = v$$

and the time interval $\frac{1}{u}$ becomes

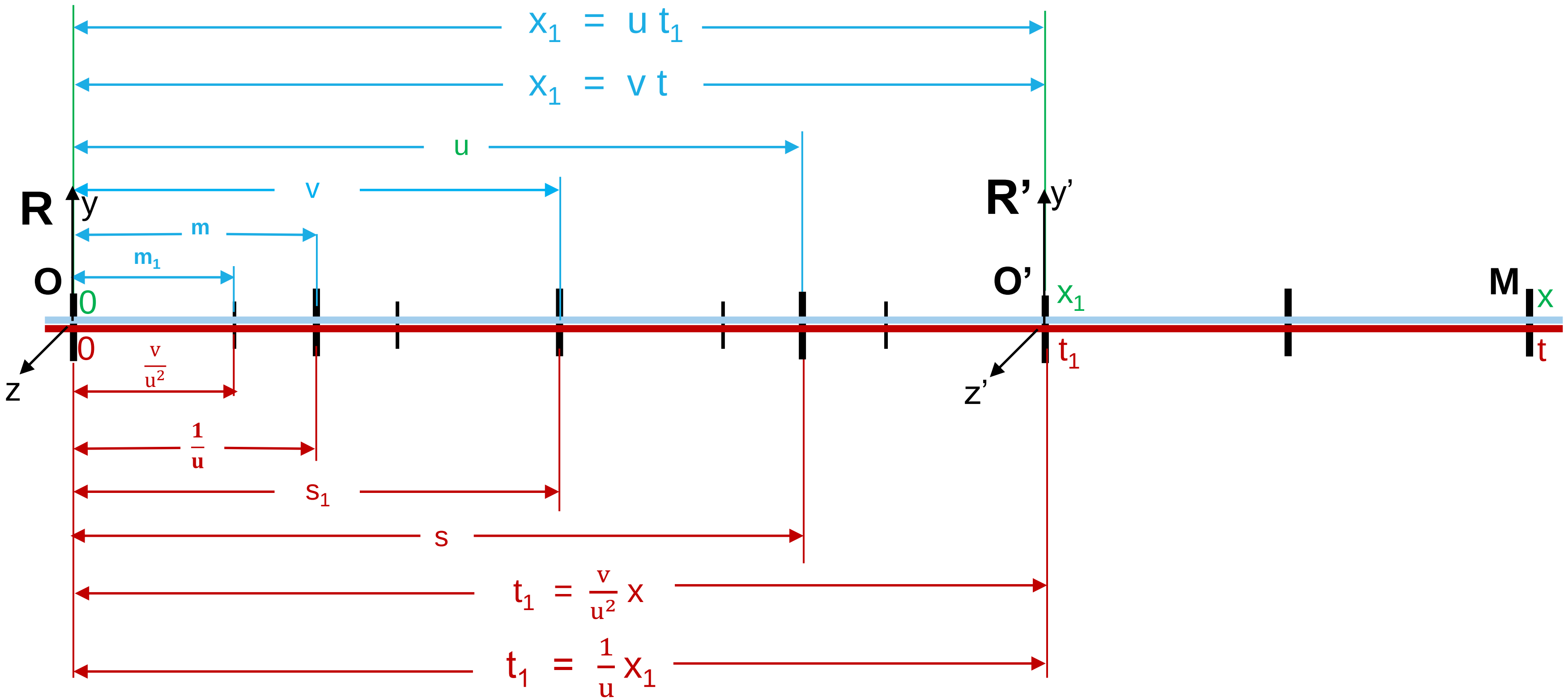
$$\alpha \frac{1}{u} = \frac{v}{u^2}$$

If we amplify the relations (1) by the factor α, it follows that the displacements of the landmark O' in relation to the landmark O on the spatial axis and on the temporal axis, respectively, in case (a) are expressed by the relations:

$$(1_1) \qquad x_1 = u\,t_1, \quad t_1 = \frac{1}{u}x_1$$

and in case (b) they are expressed by the relations:

$$(2) \qquad x_1 = v\,t, \quad t_1 = \frac{v}{u^2}\,x$$



According to (2), between the landmarks O and O' there are t distances of size v on the spatial axis (the landmark O' travels the distance x_1 in time t with speed v relative to the landmark O), respectively x time intervals of size $\frac{v}{u^2}$ on the temporal axis.

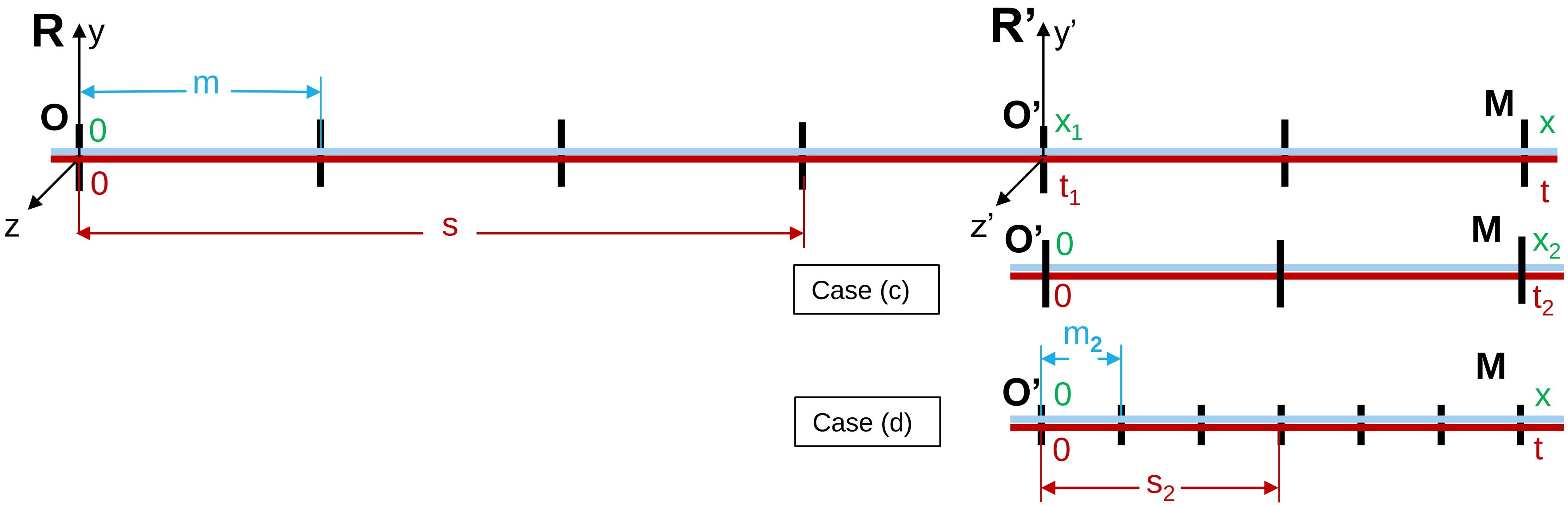
Also, the movement of the mobile M in relation to the landmark O' can be seen as a contraction of the movement of the mobile M in relation to the landmark O. In thhis case, the visual diiference betwween the movements of the mobile M in relation to the landmarks O' and O we perceive it or as a modification (contraction) of the coordinates, in which case to the mobile M are associated it the coordinates:

(c) $$x_2 = \beta x, \quad t_2 = \beta t$$

in relation to the landmark O', where β.= 1 – α, or as a change (contraction) of the units of measure, in which case the mobile M traverses the units of measure:

(d) $$m_2 = \beta m, \quad s_2 = \beta s$$

in relation to the landmark O'.



In case (d), not only units of measure, but also any distance or time interval are contracted with the β factor. Thus, the distance u becomes

$$\beta u = (1 - \alpha)\, u = u - v$$

and time intervalul $\frac{1}{u}$ becomes

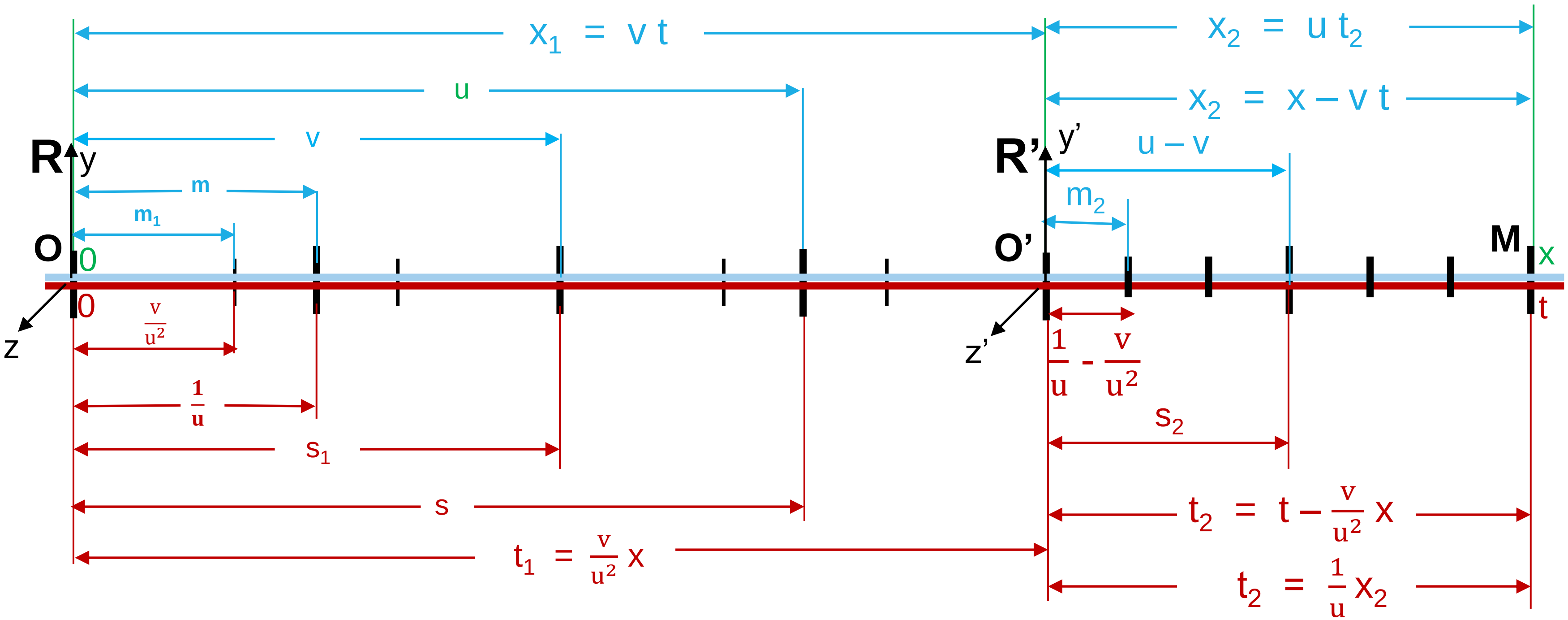
$$\beta \frac{1}{u} = (1 - \alpha) \frac{1}{u} = \frac{1}{u} - \frac{v}{u^2}$$

If we amplify the relations (1) with the factor β, it follows that the displacements of the mobile M in relation to the landmark O' on the spatial axis and on the temporal axis respectively, in case (c) are expressed by the relations:

$$x_2 = u\,t_2, \quad t_2 = \frac{1}{u}x_2 \qquad (1_2)$$

and in case (d) they are expressed by the relations:

$$x_2 = x - v\,t, \quad t_2 = t - \frac{v}{u^2}x \qquad (3)$$



According to (3), between the landmark O' and the mobile M there are t distances of size u-v on the spatial axis (the mobile M travels the distance x_2 in the time t with the speed u-v in relation to the landmark O' on the platform), respectively x time intervals of magnitude $\frac{1}{u} - \frac{v}{u^2}$ on the temporal axis.

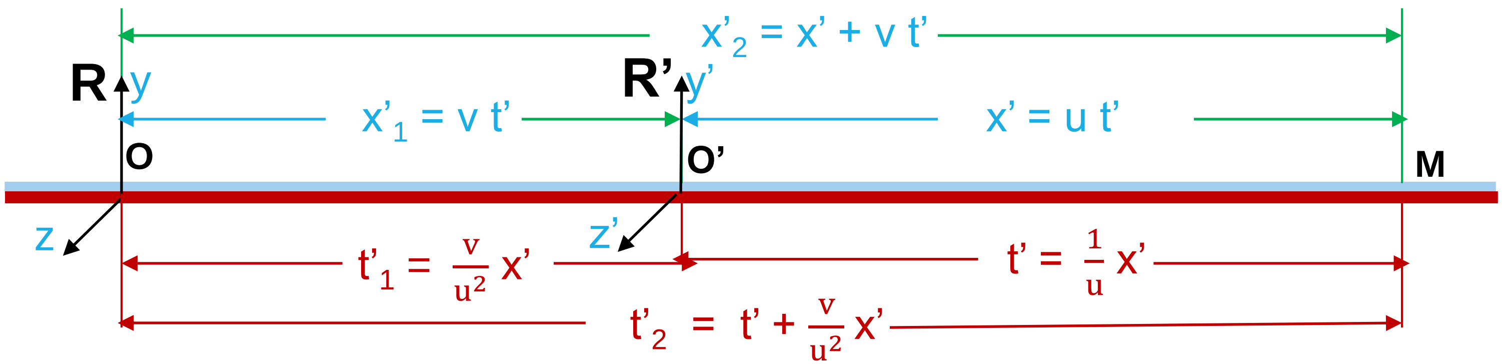
Apart from the presented case, according to which the mobile M and the landmark O' move in the same sense with the speeds u and v respectively in the referential R with the origin O, i.e. on the road, there is also a virtual case, according to which the mobile M and the landmark O moves in opposite directions with speeds u and –v, respectively, in the referential R' with the origin O', i.e. on the platform. In this case, proceeding analogously, we will deduce the formulas:

$$x' = u\,t', \quad t' = \frac{1}{u}x' \tag{1'}$$

$$x'_1 = v\,t', \quad t'_1 = \frac{v}{u^2}x' \tag{2'}$$

$$x'_2 = x' + v\,t', \quad t'_2 = t' + \frac{v}{u^2}x' \tag{3'}$$

Relations (1'), (2'), (3') describe a virtual case, that is, a possible case, but which did not happen in reality. And if this case had been real, then the case expressed by relations (1), (2), (3) would have been virtual.



Comparing the real case with the virtual one, we find that the distances and time intervals in the real case cannot be equal to the homologous distances and time intervals in the virtual case, i.e. the factor k in the equalities:

$$x = k\,(x' + v\,t'), \quad t = k\,\left(t' + \frac{v}{u^2}x'\right) \tag{4}$$

$$x' = k\,(x - v\,t), \quad t' = k\,\left(t - \frac{v}{u^2}x\right) \tag{4'}$$

it cannot be unitary. Indeed, the system of Cramer equations (4) has the solutions (4') - or conversely, the system of Cramer equations (4') has the solutions (4) - only if we assign the not unitary value to the factor k:

$$k = \frac{1}{\sqrt{1 - \frac{v^2}{u^2}}} \tag{5}$$

If u = c, where c is the speed of light in vacuum, then relations (4) and (4') are Lorentz transformations.