(i) In textbooks one can find two forms of momentum conservation equations. For example, for two-dimensional flow for *U* component of flow velocity along axis *x* these two forms can be written as

$$\rho\frac{\partial U}{\partial t}+\rho U\frac{\partial U}{\partial x}+\rho V\frac{\partial U}{\partial y}=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right) \quad (1)$$

And

$$\frac{\partial \rho U}{\partial t}+\frac{\partial}{\partial x}\rho UU+\frac{\partial}{\partial y}\rho VU=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right) \quad (2)$$

It is a general practice in computational fluid dynamics (CFD) to solve momentum equation in the form (2) for computational efficiency, stability and convergence.

Using the mass conservation equation for variable density flows

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}\rho U+\frac{\partial}{\partial y}\rho V=0$$

demonstrate that these two forms of the momentum conservation equations are equivalent.

Solution:

Applying chain rule to equation number 2

$$\frac{\partial \rho U}{\partial t}+\frac{\partial}{\partial x}\rho UU+\frac{\partial}{\partial y}\rho VU=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right)$$

$$\frac{U\partial \rho}{\partial t}+\rho\frac{\partial U}{\partial t}+\frac{U\rho U}{\partial x}+\rho U\frac{\partial U}{\partial x}+\rho U\frac{\partial U}{\partial y}+U\frac{\partial \rho V}{\partial y}$$

Taking $U\ and\ \rho$ a common

$$U\left[\frac{\partial \rho}{\partial t}+\frac{\partial \rho U}{\partial x}+\frac{\partial \rho V}{\partial y}\right]+\rho\left[\frac{\partial U}{\partial t}+\frac{U\partial U}{\partial x}+U\frac{V\partial \rho}{\partial y}\right]$$

Using the mass conservation equation for variable density flows

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}\rho U+\frac{\partial}{\partial y}\rho V=0$$

$$U\left[\frac{\partial \rho}{\partial t}+\frac{\partial \rho U}{\partial x}+\frac{\partial \rho V}{\partial y}\right]+\rho\left[\frac{\partial U}{\partial t}+\frac{U\partial U}{\partial x}+U\frac{V\partial \rho}{\partial y}\right]$$

$$0+\rho\left[\frac{\partial U}{\partial t}+\frac{U\partial U}{\partial x}+U\frac{V\partial \rho}{\partial y}\right]$$

$$\rho\left[\frac{\partial U}{\partial t}+\frac{U\partial U}{\partial x}+U\frac{V\partial \rho}{\partial y}\right]=equation\ 1$$

$$\rho\left[\frac{\partial U}{\partial t}+\frac{U\partial U}{\partial x}+U\frac{V\partial\rho}{\partial y}\right]=\frac{\partial\rho U}{\partial t}+\frac{\partial}{\partial x}\rho UU+\frac{\partial}{\partial y}\rho VU$$