PRIFYSGOL ABERTAWE
SWANSEA UNIVERSITY
College of Engineering

Example

EGA121
Introduction to Electromagnetics
LEVEL 1

University calculators permitted only

Translation dictionaries are not permitted, but an English dictionary may be borrowed from the invigilator on request.

Time allowed: **xxxx hours**

Answer the **FOUR** questions

Formulae Sheet given after the last question page.

Units

Henry H= kg m ^{2} s $^{-2}$ A $^{-2}$; Farads F= $s^4 \cdot A^2 \cdot m^{-2} \cdot kg^{-1}$

Joule J=kg m^2 s^{-2}

Important: *the student should be familiar with the calculation of capacitance of parallel plates capacitor, spherical capacitor, cylindrical capacitor and capacitors with different dielectrics inside.*

In addition, he/she should be familiar with the calculation of inductances for long solenoid, coaxial line, and toroidal coils among other structures.

The student should be also familiar with the electric and magnetic field configuration inside and outside of capacitors and inductors mentioned above.

Students should be able:

-to calculate div and curl of a vector field in cartesian, cylindrical and spherical coordinate system

- to calculate line integral for circles, straight lines and surface integrals on planes, cylinders, spheres and other high symmetry surfaces.

- to calculate volume integrals for cylindrical, spherical and rectangular volume pieces.

Q1 (covering chapter 3 of the Ulaby's book)

(a) A vector field $\vec{D} = r^3\hat{r}$ (given in cylindrical coordinate system) exist in a region between two concentric cylindrical surfaces defined by r=1 and r=2, with both cylinders extending between z=0 to z=5. Verify Divergence theorem by evaluating the following:

(i) $\oint_S \vec{D} \cdot d\vec{S}$ in the surface S enclosing the volume between the two cylinders.

[10 marks]

(ii) $\iiint_V \nabla \cdot \vec{D} dV$ over the volume V enclosed by the two cylinders.

[10 marks]

(b) Determine if the vector field $\vec{A} = \frac{\hat{R}}{R}$ (given in spherical coordinate system) is solenoidal, conservative, or both.

[5 marks]

Suggested training: Example 3.11, exercise 3.14-3.17, problems 3.47 to 3.56. example 3.12, exercise 3.18, 3.19. and problems solved in lecture slides and scan notes.

Q1. (a) Solution

(i)

$$\iint \vec{D}\cdot d\vec{s} = F_{\text{inner}} + F_{\text{outer}} + F_{\text{bottom}} + F_{\text{top}},$$

$$F_{\text{inner}} = \int_{\phi=0}^{2\pi}\int_{z=0}^{5} \left((\hat{\mathbf{r}}r^3)\cdot(-\hat{\mathbf{r}}r\,dz\,d\phi)\right)\Big|_{r=1}$$

$$= \int_{\phi=0}^{2\pi}\int_{z=0}^{5} \left(-r^4\,dz\,d\phi\right)\Big|_{r=1} = -10\pi,$$

$$F_{\text{outer}} = \int_{\phi=0}^{2\pi}\int_{z=0}^{5} \left((\hat{\mathbf{r}}r^3)\cdot(\hat{\mathbf{r}}r\,dz\,d\phi)\right)\Big|_{r=2}$$

$$= \int_{\phi=0}^{2\pi}\int_{z=0}^{5} \left(r^4\,dz\,d\phi\right)\Big|_{r=2} = 160\pi,$$

$$F_{\text{bottom}} = \int_{r=1}^{2}\int_{\phi=0}^{2\pi} \left((\hat{\mathbf{r}}r^3)\cdot(-\hat{\mathbf{z}}r\,d\phi\,dr)\right)\Big|_{z=0} = 0,$$

$$F_{\text{top}} = \int_{r=1}^{2}\int_{\phi=0}^{2\pi} \left((\hat{\mathbf{r}}r^3)\cdot(\hat{\mathbf{z}}r\,d\phi\,dr)\right)\Big|_{z=5} = 0.$$

Therefore, $\iint \vec{D}\cdot d\vec{s} = 150\pi$.

(ii)

From the back cover, $\nabla\cdot\vec{D} = (1/r)(\partial/\partial r)(rr^3) = 4r^2$. Therefore,

$$\iiint \nabla\cdot\vec{D}\,d\mathscr{V} = \int_{z=0}^{5}\int_{\phi=0}^{2\pi}\int_{r=1}^{2} 4r^2\,r\,dr\,d\phi\,dz = \left(\left(\left(r^4\right)\Big|_{r=1}^{2}\right)\Big|_{\phi=0}^{2\pi}\right)\Bigg|_{z=0}^{5} = 150\pi.$$

(b) Solution, the field A is conservative as its curl is zero, please verify by using the curl formula in spherical.

Q2 (covering chapter 4 of Ulaby's book)

The parallel plate capacitor shown in figure Q2 is formed by two perfect metal plates of area *A* separated by a distance *d*, the space between the plates contains two adjacent dielectrics, one with permittivity ε_1 and area A_1 and another with ε_2 and A_2. An applied voltage *V* between the plates develops a charge +*Q* in the top plate and a charge -*Q* in the bottom plate.

(i) Derive an expression for the electric field in the dielectric. *Hint: Make reasonable approximations for the field and use Gauss's law.*

[9 marks]

(ii) Derive an expression for the electrical potential V between the plates as a function of Q, ε1, ε2, d, A1 and A2. Use the expression of the electric field derived from (i).

[6 marks]

(iii) Derive an expression for the capacitance as function of ε1, ε2, d, A_1 and A_2.

[6 marks]

(iv) Calculate the electrostatic energy inside the capacitor if *V,* ε1, ε2, d, A1 and A2 are equal to 2V, $10\varepsilon_0$, $3\varepsilon_0$ 1mm, 0.3cm^2 0.5cm^2 respectively.

[4 marks]

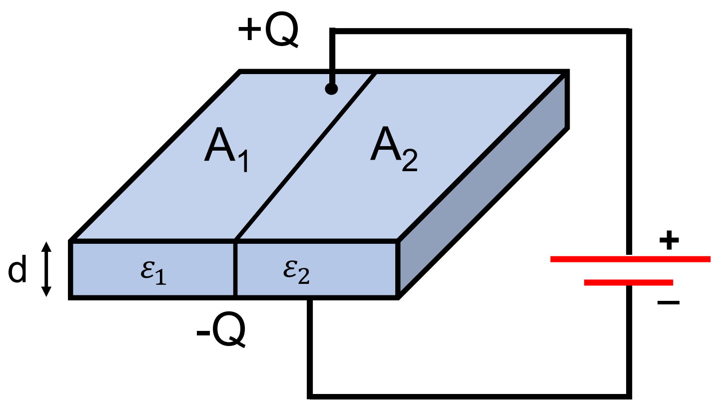


Figure Q2. Parallel plate capacitor with two different dielectrics between the plates.

Suggested training:
Exercises and examples in the lecture notes (slides + Scan)
Problems from book
Capacitance: 4.55,4.56,4.57,4.58,4.60
Boundary Conditions D,E: 4.48,4.49,4.50, 4.51
Electrical potential: 4.32,4.36
Gauss's Law 4.23, 4.24, 4.25, 4.26, 4.27, 4.28, 4.29.

Q3 (covering chapter 5 of Ulaby's book)

1. A solenoid (Figure Q3.1) of length *c* and circular cross section A has a core of non-conductive material of permeability μ. A current *I* flows through the winding as indicated in figure Q3.1. Assume that the length c of the solenoid is much longer than the radius and that the number of turns is *N*.

   (i) Derive an expression of the magnetic field B inside the solenoid as a function of the current *I* and *N*. *Use Ampere's law and reasonable arguments to simplify the expression of the H field.*

   [5 marks]

   (ii) Calculate the flux of B, $\Phi = \int_S \vec{B} \cdot d\vec{S}$ through a cross section of the core through a rectangular area shown in Figure Q3.

   [10 marks]

   (iii) Derive an expression for the inductance *L* as a function of *A*, *c* and μ.

   [5 marks]

   (iv) Calculate the magnetic energy stored inside the solenoid if *I, A, c, N* and μ are equal to *1A*, *2cm*^{2}, *100cm*, *100* and $3\mu_0$ respectively.

   [5 marks]

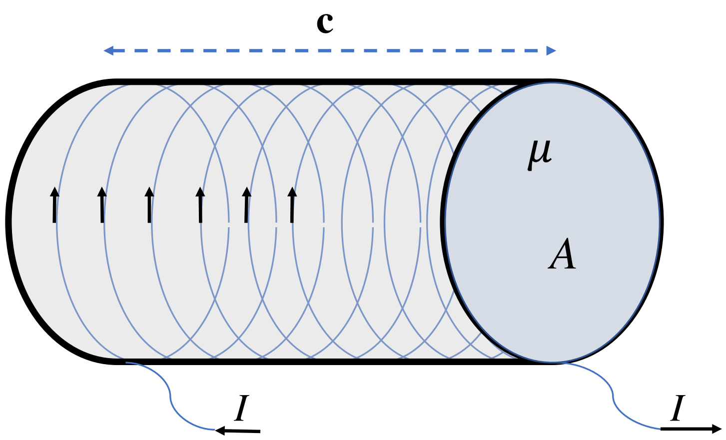


Figure Q3.1 Solenoid with core of permeability μ, area *A* and *N* turns. The arrows indicate the current direction.

Suggested training to similar questions (chapter 5):
Problems from book

Ampere's law: 5.21, 5.22, 5.23.
Boundary conditions H, B: 5.32, 5.33
Inductance and magnetic energy: 5.35,5.36,5.37,5.38, 5.39, 5.40

Q4 (covering chapter 5 of Ulaby's book)

(a) In a non-conducting medium with $\varepsilon = 2\varepsilon_0$ and $\mu = 3\mu_0$, the electric field intensity of an electromagnetic wave is given by: $\vec{E}(z,t) = E_0 \sin(\omega t - kz - \pi)\,\hat{x}$

(i) Determine the magnetic flux $\vec{B}(z,t)$. *Hint: convert E to phasor notation and then use the appropriate phasor equation given in the formulae sheet to find B*

[15 marks]

(ii) Express k (wavenumber) as a function of ω and c (the speed of light).

[5 marks]

(ii) Sketch the diagram of the plane polarized electromagnetic wave of (i) at $t=0$ and indicate the direction of propagation. Show the electric and magnetic field direction.

[5 marks]

Q4 (extra question)

The electric field radiated by a short dipole antenna is given in spherical coordinates by:
$\vec{E}(r,\theta,t) = \frac{E_0 \sin(\theta)}{R} \sin(\omega t - 2\pi R)\,\hat{\theta}$

(iii) Determine the magnetic field $\vec{H}(R,\theta,t)$. *Hint: convert E to phasor notation and then use the appropriate phasor equation given in the formulae sheet to find B.*

[20 marks]

(ii) Sketch the electrical and magnetic field of this electromagnetic wave.

[5 marks]

Suggested training to this type of question (chapter 6):

Exercises and examples in the lecture notes (slides + Scan)
Problems from book
Faraday's Law: 6.1, 6.2, 6.3,6.4,6.5, 6.8, 6.9, 6.10
Displacement current: 6.15, 6.16
Electromagnetic potential: 6.23,6.24, 6.25, 6.26, 6.27, 6.28

Other type of exam questions

Q2 Electrostatic

The figure Q2 shows three charged metal spheres. The charges in each of them is given in units of Q, Q is assumed positive.
(i) Sketch the electric field E around the charges. Indicate where are the sources or sink of electric field, or where the divergence of the electric field is positive, negative or zero.

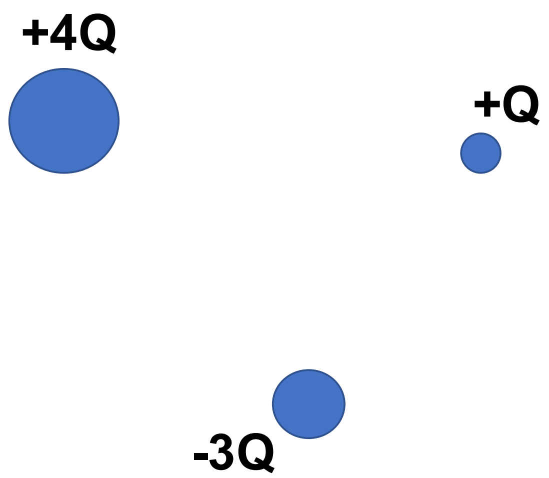


Figure Q2. Three charges in the plane. The magnitude of the charges is given in units of Q. (Q is a positive value).
Solution: The divergence of E is positive in the surface of the positive charges, negative in the surface of the negative charge and zero in free space where there are no charges.

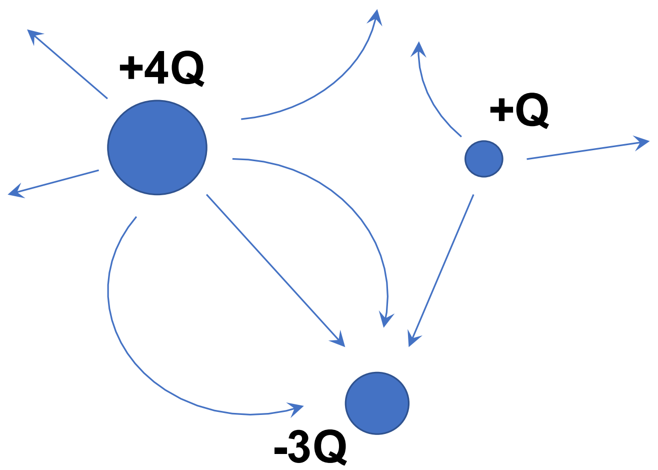


Figure Q2 with the sketch of the field

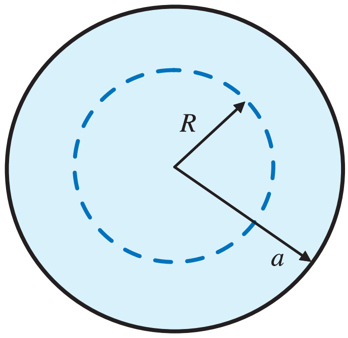
Q2. Electrostatic

Exercise 4.9: A spherical volume of radius *a* contains a uniform volume charge density ρ_V. Use Gauss's law to determine the vector **D** for (a) $R<a$ and (b) $R>a$.

In the figures that follows, the dashed circles are the chosen Gaussian spheres to calculate **D** for (a) and (b)

Solution:

(a)



$R < a$

For $R \leq a$,

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = \oint_S D_r \, ds = D_r(4\pi R^2)$$

Q within a sphere of radius R is

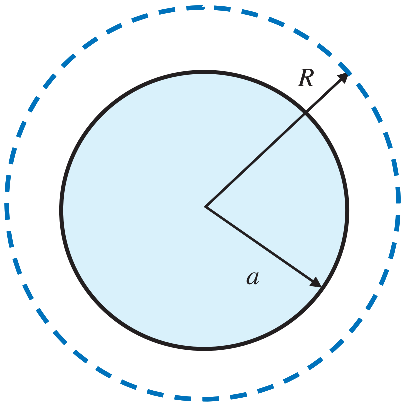
$$Q = \frac{4}{3}\pi R^3 \rho_v$$

Hence,

$$4\pi R^2 D_R = \frac{4}{3}\pi R^3 \rho_v$$

$$D_r = \frac{\rho_v R}{3}, \qquad \mathbf{D} = \hat{\mathbf{R}} D_r = \hat{\mathbf{R}} \frac{\rho_v R}{3}, \qquad R \leq a.$$

(b)

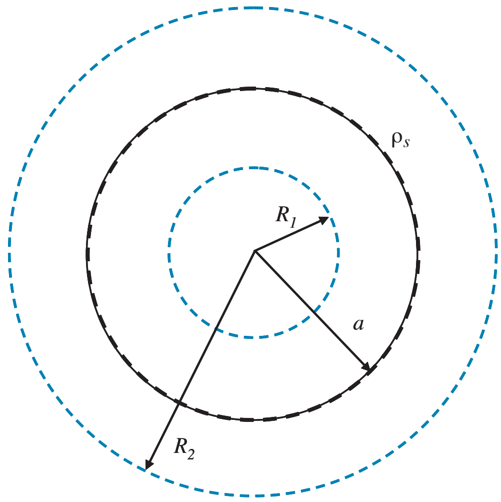


$R > a$

Q2. Electrostatic

Exercise 4.8 A thin spherical shell of radius a carries a uniform surface charge density ρ_s. Use Gauss's law to determine $\mathbf{E}$.

Solution:



$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$$

Symmetry suggests that $\mathbf{D}$ is radial in direction. Hence,

$$\mathbf{D} = \hat{\mathbf{R}} D_R$$
$$d\mathbf{s} = \hat{\mathbf{R}}\, ds$$
$$\oint_S \mathbf{D} \cdot d\mathbf{s} = \oint_S D_R\, ds = D_R(4\pi R^2) = Q$$
$$D_R = \frac{Q}{4\pi R^2}$$

- For a Gaussian surface of radius $R_1 < a$, no charge is enclosed. Hence, $Q = 0$, in which case $E = 0$.
- For a Gaussian surface of radius $R_2 > a$,

$$Q = \rho_s(4\pi a^2)$$

and

$$\mathbf{E} = \frac{\mathbf{D}}{\varepsilon} = \frac{\hat{\mathbf{R}}}{\varepsilon} D_r = \frac{\hat{\mathbf{R}} Q}{4\pi\varepsilon R_2^2} = \hat{\mathbf{R}} \frac{4\pi\rho_s a^2}{4\pi\varepsilon R_2^2} = \hat{\mathbf{R}} \frac{\rho_s a^2}{\varepsilon R_2^2}\,.$$

Q3. Magnetostatic

(a) For each of the following contours denoted by C1, C2 , C3, C4 evaluate the $\oint_C \vec{H} \cdot \vec{dl}$ using Ampere's law, ⊗ represents a current going into the page, consequently ⊙ represents going out of the page. The magnitude of the current is indicated in units of *I* close to the corresponding current. Note that the sign (+ or -) of the answer depends on the orientation of the contour. *Hint: use the hand right rule*.

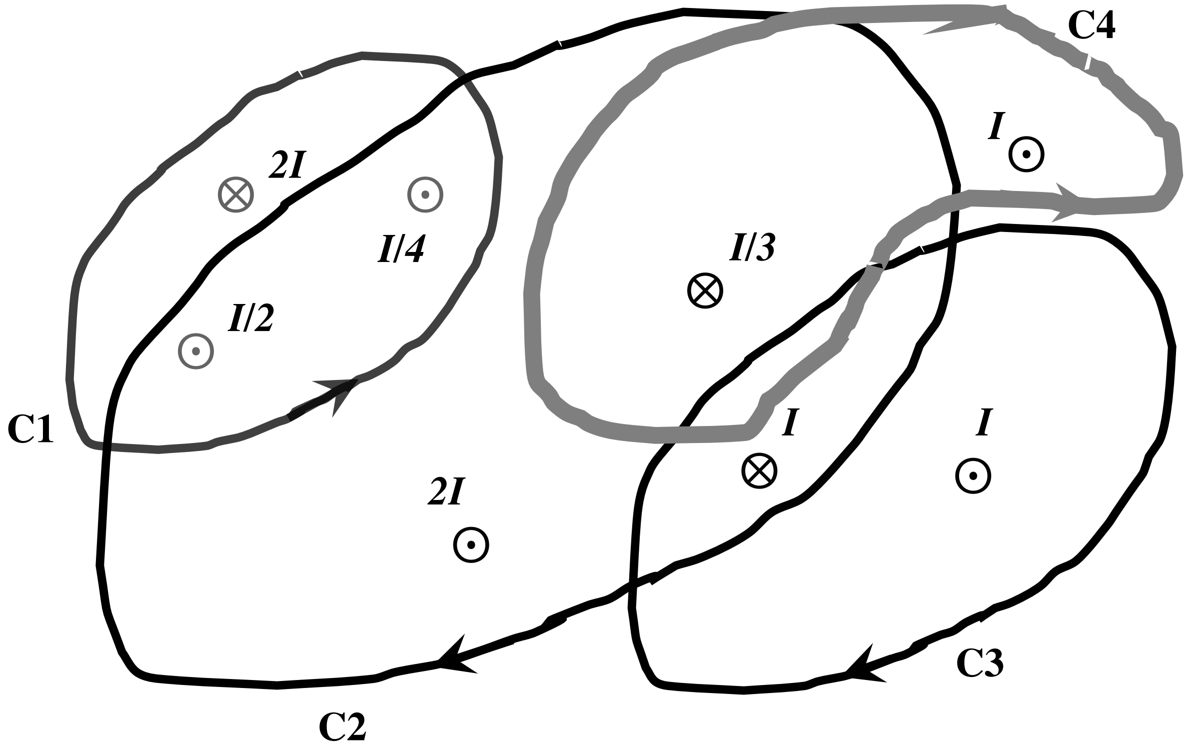


Figure Q3.1 Contours C1, C2, C3 and C4 enclosing currents entering and leaving the page.

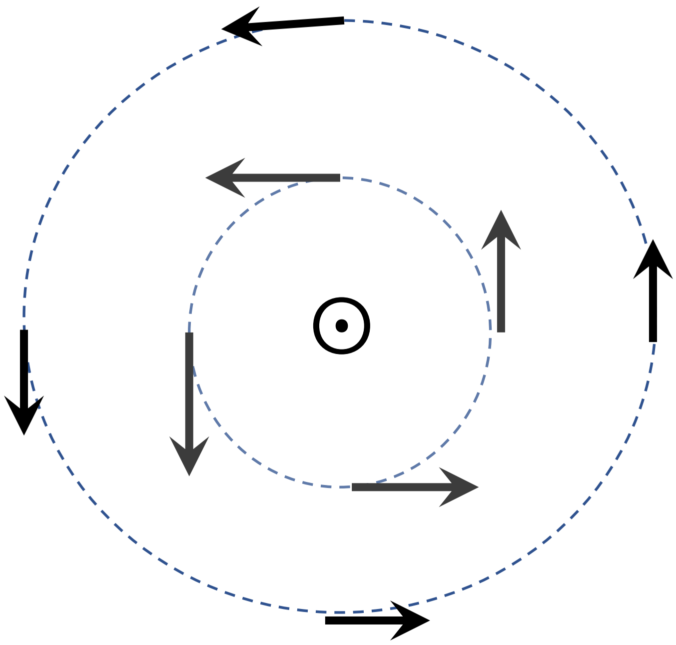
Solution: C1: .-(5/4)I; C2: -I(1/2+1/4+2-1-1/3), C3:0, C4:I(1-1/3)

(b) The infinite straight wire perpendicular to the page shown carries a current *I* pointing out of the page.

(i) Sketch the magnetic field $\vec{H}$ in the vicinity of the wire.

(ii) Derive an expression for the magnetic field $\vec{H}$ as a function of the distance *r* from the wire and the current *I* in the wire. *Hint: make reasonable approximation for the field and use Ampere's law*. ***Solution in the lecture notes and scan***

Solution (i)



H field is tangent to the circles with centre in the wire and follows the right-hand rule.

Q3. Example 5.3 Magnetic field of a long wire.
A long (practically infinite) straight wire of radius a carries a steady current I that is uniformly distributed over its cross section. Determine the magnetic field $\boldsymbol{H}$ a distance r from the wire axis for (i) $r<a$ (inside the wire) and (ii) $r>a$ (outside the wire)

Solution: in the lectures slides.

Q4 this question is very similar that the extra question Q4

6.26 The electric field radiated by a short dipole antenna is given in spherical coordinates by

$$\mathbf{E}(R,\theta;t) = \hat{\boldsymbol{\theta}}\,\frac{2\times 10^{-2}}{R}\sin\theta\,\cos(6\pi\times 10^{8}t - 2\pi R) \quad \text{(V/m)}.$$

Find $\mathbf{H}(R,\theta;t)$.

Solution: Converting to phasor form, the electric field is given by

$$\tilde{\vec{E}}(R,\theta) = \hat{\boldsymbol{\theta}}E_\theta = \hat{\boldsymbol{\theta}}\frac{2\times 10^{-2}}{R}\sin\theta\exp - j2\pi R \qquad \text{(V/m)},$$

which can be used with Eq. (6.87) to find the magnetic field:

$$\begin{aligned}
\tilde{\vec{H}}(R,\theta) &= \frac{1}{-j\omega\mu}\nabla\times\tilde{\vec{E}} = \frac{1}{-j\omega\mu}\left[\hat{\mathbf{R}}\frac{1}{R\sin\theta}\frac{\partial E_\theta}{\partial\phi} + \hat{\boldsymbol{\phi}}\frac{1}{R}\frac{\partial}{\partial R}(RE_\theta)\right] \\
&= \frac{1}{-j\omega\mu}\hat{\boldsymbol{\phi}}\frac{2\times 10^{-2}}{R}\sin\theta\frac{\partial}{\partial R}(\exp - j2\pi R) \\
&= \hat{\boldsymbol{\phi}}\frac{2\pi}{6\pi\times 10^{8}\times 4\pi\times 10^{-7}}\frac{2\times 10^{-2}}{R}\sin\theta\exp - j2\pi R \\
&= \hat{\boldsymbol{\phi}}\frac{53}{R}\sin\theta\,\exp - j2\pi R \qquad (\mu\text{A/m}).
\end{aligned}$$

Converting back to instantaneous value, this is

$$\vec{H}(R,\theta;t) = \hat{\boldsymbol{\phi}}\frac{53}{R}\sin\theta\cos\left(6\pi\times 10^{8}t - 2\pi R\right) \qquad (\mu\text{A/m}).$$

END OF QUESTIONS

Formulae Sheet

FUNDAMENTAL PHYSICAL CONSTANTS		
CONSTANT	SYMBOL	VALUE
speed of light in vacuum	c	$2.998 \times 10^8 \simeq 3 \times 10^8$ m/s
gravitational constant	G	6.67×10^{-11} N·m^2/kg^2
Boltzmann's constant	K	1.38×10^{-23} J/K
elementary charge	e	1.60×10^{-19} C
permittivity of free space	ε_0	$8.85 \times 10^{-12} \simeq \frac{1}{36\pi} \times 10^{-9}$ F/m
permeability of free space	μ_0	$4\pi \times 10^{-7}$ H/m
electron mass	m_e	9.11×10^{-31} kg
proton mass	m_p	1.67×10^{-27} kg
Planck's constant	h	6.63×10^{-34} J·s
intrinsic impedance of free space	η_0	$376.7 \simeq 120\pi\ \Omega$

Gradient of a scalar field V in Cartesian coordinates:

$$\nabla V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}$$

The Laplacian ∇^2 of a scalar quantity V is given in Cartesian coordinates by:

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

For a vector field $\vec{A}$ in Cartesian coordinates, given in component form as: $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$ we have the following expressions for the divergence and the curl:

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{z}$$

The Laplacian ∇^2 of a vector field $\vec{A}$ is given in Cartesian coordinates by:

$$\nabla^2 \vec{A} = \frac{\partial^2 A_x}{\partial x^2}\hat{x} + \frac{\partial^2 A_y}{\partial y^2}\hat{y} + \frac{\partial^2 A_z}{\partial z^2}\hat{z}$$

(4) Vector calculus identity for a vector field $\vec{A}$:

$$\nabla \times \left(\nabla \times \vec{A}\right) = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla^2 \vec{A}$$

Gradient of a scalar field V in cylindrical coordinates:

$$\nabla V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{\phi} + \frac{\partial V}{\partial z}\hat{z}$$

For a vector field $\vec{A}$ in cylindrical coordinates, given in component form as:
$\vec{A} = A_r\hat{r} + A_\phi\hat{\phi} + A_z\hat{z}$, we have the following expressions for the divergence and the curl:

$$\nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial}{\partial \phi}A_\phi + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\phi} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right)\hat{z}$$

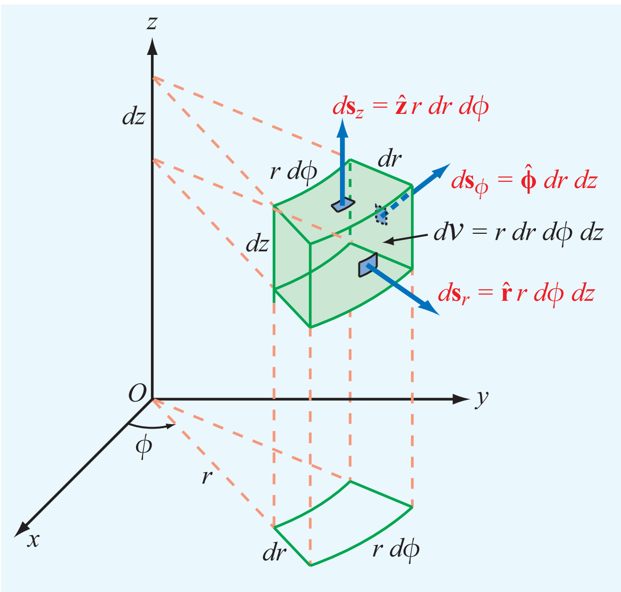


Fig. 1. Differential areas and volume in cylindrical coordinates.

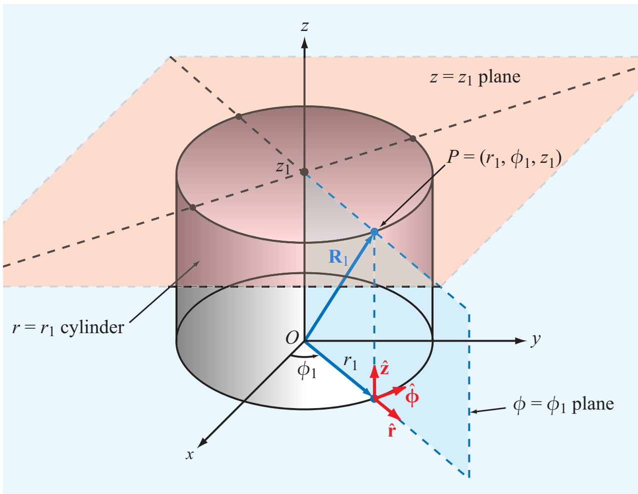


Fig. 2. Coordinate unit vectors in cylindrical. Coordinate system. Surfaces, in which one of the coordinates is constant, are also shown.

Gradient of a scalar field V in spherical coordinates:

$$\nabla V = \frac{\partial V}{\partial R}\hat{R} + \frac{1}{R}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}$$

For a vector field $\vec{A}$ in spherical coordinates, given in component form as:
$\vec{A} = A_R\hat{R} + A_\theta\hat{\theta} + A_\phi\hat{\phi}$, we have the following expressions for the divergence and the curl:

$$\nabla \cdot \vec{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{Rsin\theta}\left(\frac{\partial}{\partial \theta}(A_\phi sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right)\hat{R} + \frac{1}{R}\left(\frac{1}{sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right)\hat{\theta}$$
$$+ \frac{1}{R}\left(\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right)\hat{\phi}$$

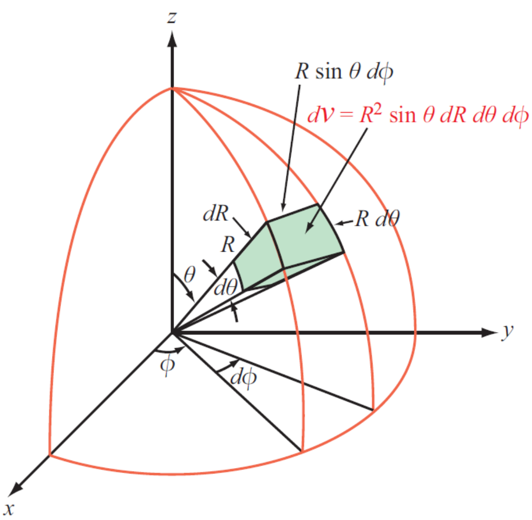


Fig. 1. Spherical coordinate system showing line and volume elements

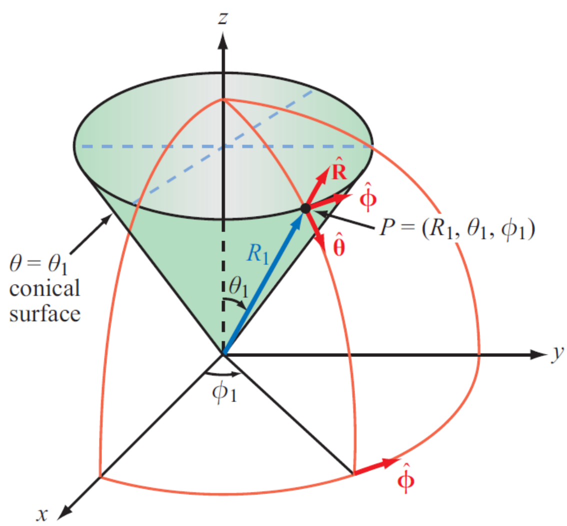


Fig. 2. Coordinate unit vectors in spherical coordinates

Table 3-1: Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\lvert\mathbf{A}\rvert =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}} \cdot \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}} \cdot \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}} \cdot \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \cdot \hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}} \times \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\mathbf{A} \cdot \mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A} \times \mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Table 3-2: Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

Divergence Theorem converts the integration over a volume to one over the surface enclosing that volume and vice versa.

$$\int_V (\nabla \cdot \vec{E})\, dV = \oint_S \vec{E} \cdot \vec{dS}$$

Stokes's theorem converts the surface integral of the curl of a vector field on an open surface S, into a line integral of the vector field along the contour C bordering the surface S.

$$\int_S (\nabla \times \vec{B}) \cdot \vec{dS} = \oint_C \vec{B} \cdot \vec{dl}$$

Maxwell's Equations (point form)

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Maxwell's Equations (integral Form)

$$\oint_S \vec{D} \cdot \overrightarrow{dS} = Q$$

$$\oint_C \vec{E} \cdot \overrightarrow{dl} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \overrightarrow{dS}$$

$$\oint_S \vec{B} \cdot \overrightarrow{dS} = 0$$

$$\oint_C \vec{H} \cdot \overrightarrow{dl} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \cdot \overrightarrow{dS}$$

With

$$\vec{D} = \varepsilon \vec{E}$$
$$\vec{B} = \mu \vec{H}$$

In vacuum: $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$

Boundary conditions for $\vec{E}$ and $\vec{D}$ at the surface between two dielectrics:

$D_{1n} - D_{2n} = \rho_S$ where ρ_S is the free surface charge (excluding polarization charges)

$E_{1T} = E_{2T}$

Boundary conditions for $\vec{B}$ and $\vec{H}$ at the surface between two magnetic materials:

$H_{1T} - H_{2T} = J_S$ where J_S is the free surface current (excluding magnetizing currents)

$B_{1n} = B_{2n}$

The electrical potential difference between a and b is given by:

$V_{ab} = V_a - V_b = -\int_b^a \vec{E} \cdot \overrightarrow{dl}$

Resistance

$$R = \frac{V}{I} = \frac{-\int_l \vec{E} \cdot \overrightarrow{dl}}{\int_s \vec{J} \cdot \overrightarrow{dS}} = \frac{-\int_l \vec{E} \cdot \overrightarrow{dl}}{\int_s \sigma\vec{E} \cdot \overrightarrow{dS}}$$

Capacitance

$$C = \frac{Q}{V} = \frac{\int_s \vec{E} \cdot \overrightarrow{dS}}{-\int_l \vec{E} \cdot \overrightarrow{dl}}$$

Energy stored in a capacitor

$$W_e = \frac{1}{2} C V^2$$

Energy stored in inductor

$$W_m = \frac{1}{2} L I^2$$

Inductance L,

$$L = \frac{\Lambda}{I}$$

Where Λ is the magnetic flux linkage and I the current.

Euler's identity

$$e^{j\omega t} = cos(\omega t) + j \sin(\omega t)$$

A vector $\vec{\boldsymbol{A}}(z,t) = \vec{\boldsymbol{b}}(z) \cos(\omega t + \phi(z))$ can be written in phasor notation as:

$$\vec{\boldsymbol{A}}(z,t) = real\left(\tilde{\boldsymbol{b}}(\boldsymbol{z})\, e^{j\omega t}\right)$$

where $\tilde{\boldsymbol{b}}(\boldsymbol{z}) = \vec{\boldsymbol{b}}(\boldsymbol{z}) e^{j\phi(z)}$ is the phasor, $\phi(z)$ is a function of z and $\vec{\boldsymbol{b}}(\boldsymbol{z})$ a real vector.

Maxwell's equations for phasors

$$\nabla \times \tilde{H} = j\omega\varepsilon\tilde{E}$$

$$\nabla \times \tilde{E} = -j\omega\mu\tilde{H}$$

Speed of light as a function of permeability μ_0 and permittivity ε_0 of the vacuum

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Derivatives and Integrals (all the integrals are defined up to an additive constant)

$$\frac{d}{dx} x^n = n x^{(n-1)}$$

$$\frac{d}{dx} \sin(x) = \cos(x)$$

$$\frac{d}{dx} \cos(x) = -\sin\,(x)$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$$

$$\int \sin(x)\, dx = -\cos\,(x)$$

$$\int \cos\,(\mathrm{x}) dx = \sin(x)$$

$$\int \cos(\mathrm{x}) \sin(x)\, dx = \frac{(\sin(x))^2}{2}$$

$$\int \frac{1}{x} dx = \ln\,(x)$$

Area and volume of sphere

$$A = 4\pi r^2 \qquad V = \frac{4\pi r^3}{3}$$

Sine and cosine table angles

x	0	π/6	π/4	π/3	π/2	π	3π/2	2π
sin(x)	0	1/2	1/ √2	√3 / 2	1	0	-1	0
cos(x)	1	√3 / 2	1/ √2	1/2	0	-1	0	1

$$\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$$