PRIFYSGOL ABERTAWE
SWANSEA UNIVERSITY
College of Engineering

Take-Home Exam

May 2022

EGA121

Introduction to Electromagnetics

Year 1

Deadline: 25/May/10am

Time allowed: **24 Hours**
Answer all **FOUR** questions
Show your work and box the final answer.

Formulae sheet provided from page 6 to 14.

- This is an individual assessment; working in groups, helping or receiving help from other students or third parties is not permitted. **The work you submit must be your own work.** The usual University regulations regarding academic misconduct apply. For further information regarding Academic Misconduct, please follow the links:

  https://myuni.swansea.ac.uk/academic-life/academic-regulations/assessment-and-progress/academic-misconduct-procedure/
  https://myuni.swansea.ac.uk/media/AM-Code-of-Practice-2018-19-FINAL.pdf

TURN OVER

Q1. Vector Calculus

(a) Given the vector fields $\vec{G} = 2y\hat{x} + 2x\,\hat{y} - z^2\hat{z}$ in cartesian coordinates and $\vec{F} = \hat{\phi}$ in cylindrical coordinates. Determine whether these vector fields are solenoidal, conservative, both solenoidal and conservatives or neither of these.

[5 marks]

(b) For the vector field $\vec{A} = 3\phi\hat{\phi} + 2\hat{r}$ given in cylindrical coordinate system.

(i) Calculate $\nabla \times \vec{A}$. [5 marks]

(ii) Calculate the line integral of $\vec{A}$ ($\int_C \vec{A} \cdot \overrightarrow{dl}$) in the contour ***C*** shown in figure Q1. The contour is traverse in the counter-clockwise direction.

[15 marks]

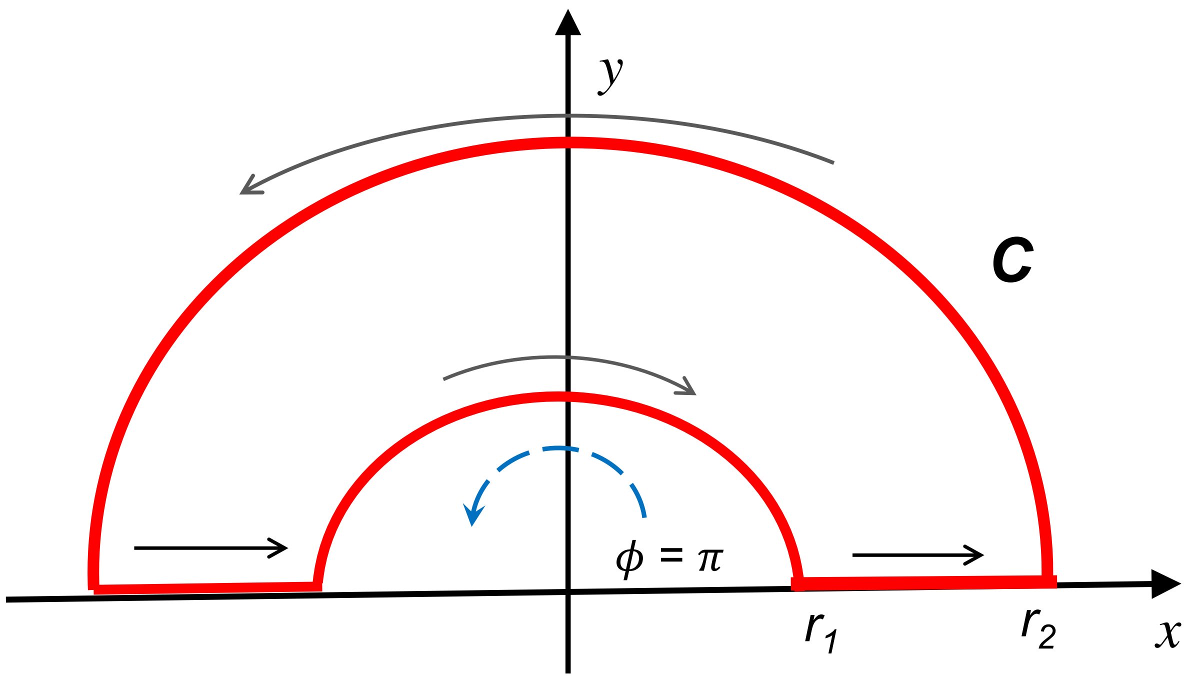

Figure Q1. The contour C (in red) is formed by two semicircles plus two horizontal segments of length r2-r1 and it is traverse counter-clockwise as indicated in the figure. The semicircles span a 90 degrees angle (Pi in radians).

(Total 25 marks)

TURN OVER

Q2. Electrostatics.

The spherical capacitor shown in Figure Q2 is formed by two concentric spherical conducting shells, the inner with radius *a* and outer with radius *b*. The capacitor is filled with two dielectrics of permittivities ε_1 and ε_2. The dielectric with permittivity ε_1 fills 1/4 of the volume as shown in Figure Q2, the remaining region is filled by the other dielectric. An applied voltage V between the spheres produces a charge +Q in the inner sphere and a charge -Q in the outer sphere.

(a) Derive expressions for the electric field $\vec{E}$ and displacement field $\vec{D}$ in each dielectric as a function of Q, ε_1 and ε_2. Assume that both fields are radial. Hint: *Make use of Gauss's law and the boundary conditions for* $\vec{E}$ and $\vec{D}$.

[15 marks]

(b) Derive an expression for the capacitance as a function of *a, b,* ε_1 and ε_2.

[10 marks]

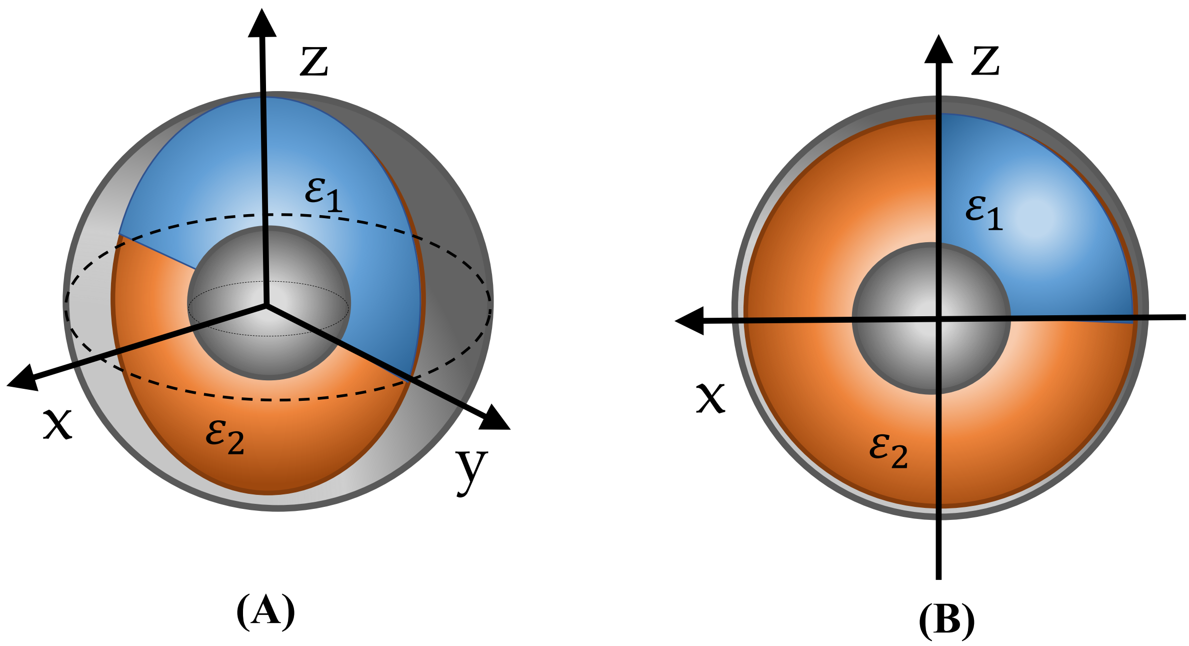

Figure Q2. (A) Spherical capacitor with two dielectrics. The capacitor is filled with dielectrics of permittivities ε_1 and ε_2 as shwn in the figure. The dielectric of permittivity ε_1 fills a quarter of the volume between the shells. The remaining region is filled by dielectric ε_2. (B) Depicts a x-z plane (z=0) view of the capacitor showing the different dielectric filling.

(Total 25 marks)

Q3. Magnetostatics.

The coaxial inductor shown in figure Q3.1 is formed by two concentric conducting cylinders of length *L*. The inner cylinder has a radius a and the outer cylinder a radius b. The inductor is filled with three non-conducting magnetic materials of permeability μ_1, μ_2 and μ_3. The material of permeability μ_3 fills half of the volume and materials μ_1 and μ_2 one quarter of the volume as indicated in the cross section in figure Q3.1. If a current *I* circulates in the outer conductor from left to right and in the inner conductor from right to left:

(a) Derive expressions for the "magnetic" field vectors $\vec{B}$ and $\vec{H}$ in the three materials inside the inductor as a function of the distance *r* from the axis, the permeabilities and the current *I*. Assume that the fields are axials (in the direction of the cylindrical vector $\hat{\phi}$). *Hint: Make use of the boundary condition for* $\vec{B}$, $\vec{H}$ *and Ampere's law.*

[15 marks]

(b) Derive an expression for the inductance as a function of a, *b*, *L*, μ_1, μ_2 and μ_3 by using the expression of the magnetic field $\vec{B}$ calculated in (a).

[10 marks]

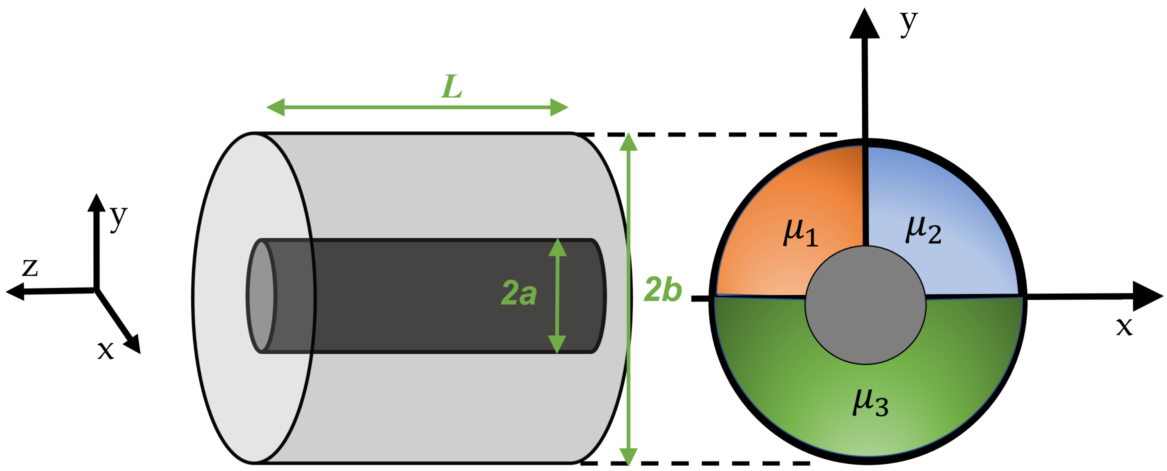

Figure Q3.1. Coaxial inductor indicating geometry, dimension and cross section.

(Total 25 marks)

TURN OVER

Q4. Electromagnetic Waves.

In a non-conducting medium with $\varepsilon = \frac{9\varepsilon_0}{5}$ and $\mu = 5\mu_0$, the magnetic field of a plane polarized electromagnetic wave is given by: $\vec{E}(z,t) = E_0 sin(\omega t - kz + \pi)\hat{y}$

(a) Determine the phasor $\tilde{E}(z)$ of $\vec{E}(z,t)$.

[5 marks]

(b) Determine the phasor $\widetilde{H}(z)$ using Maxwell's phasor equations.

[9 marks]

(c) Determine the magnetic field $\vec{H}(z,t)$.

[5 marks]

(d) Express *k* (wavenumber) as a function of ω and *c* (the speed of light).

[1 mark]

(e) Sketch a diagram of the electromagnetic wave at *t=0* and indicate the direction of propagation. Show the electric and magnetic field direction.

[5 marks]

(Total 25 marks)

END OF EXAM

Formulae Sheet

FUNDAMENTAL PHYSICAL CONSTANTS		
CONSTANT	SYMBOL	VALUE
speed of light in vacuum	c	$2.998 \times 10^8 \simeq 3 \times 10^8$ m/s
gravitational constant	G	6.67×10^{-11} N·m^2/kg^2
Boltzmann's constant	K	1.38×10^{-23} J/K
elementary charge	e	1.60×10^{-19} C
permittivity of free space	ε_0	$8.85 \times 10^{-12} \simeq \frac{1}{36\pi} \times 10^{-9}$ F/m
permeability of free space	μ_0	$4\pi \times 10^{-7}$ H/m
electron mass	m_e	9.11×10^{-31} kg
proton mass	m_p	1.67×10^{-27} kg
Planck's constant	h	6.63×10^{-34} J·s
intrinsic impedance of free space	η_0	$376.7 \simeq 120\pi$ Ω

Gradient of a scalar field V in Cartesian coordinates:

$$\nabla V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}$$

The Laplacian ∇^2 of a scalar quantity V is given in Cartesian coordinates by:

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

For a vector field $\vec{A}$ in Cartesian coordinates, given in component form as:
$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$ we have the following expressions for the divergence and the curl:

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{z}$$

The Laplacian ∇^2 of a vector field $\vec{A}$ is given in Cartesian coordinates by:

$$\nabla^2 \vec{A} = \frac{\partial^2 A_x}{\partial x^2}\hat{x} + \frac{\partial^2 A_y}{\partial y^2}\hat{y} + \frac{\partial^2 A_z}{\partial z^2}\hat{z}$$

(4) Vector calculus identity for a vector field $\vec{A}$:

$$\nabla \times \left(\nabla \times \vec{A}\right) = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla^2 \vec{A}$$

Gradient of a scalar field V in cylindrical coordinates:

$$\nabla V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{\phi} + \frac{\partial V}{\partial z}\hat{z}$$

For a vector field $\vec{A}$ in cylindrical coordinates, given in component form as: $\vec{A} = A_r\hat{r} + A_\phi\hat{\phi} + A_z\hat{z}$, we have the following expressions for the divergence and the curl:

$$\nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial}{\partial \phi}A_\phi + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\phi} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right)\hat{z}$$

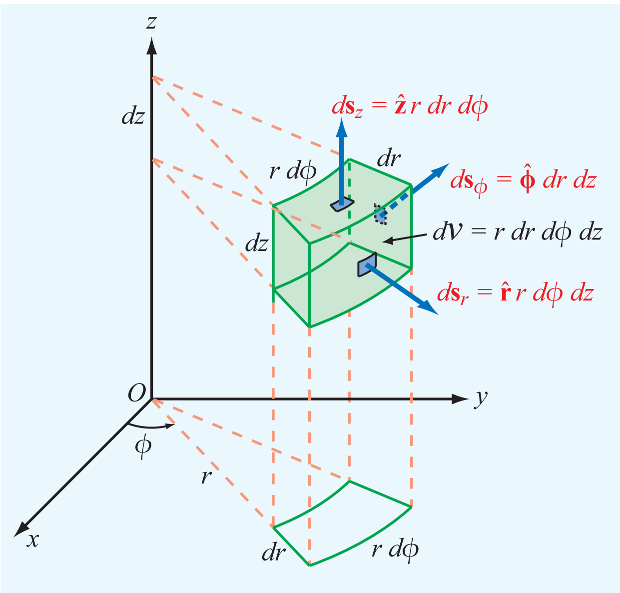

Fig. 1. Differential areas and volume in cylindrical coordinates.

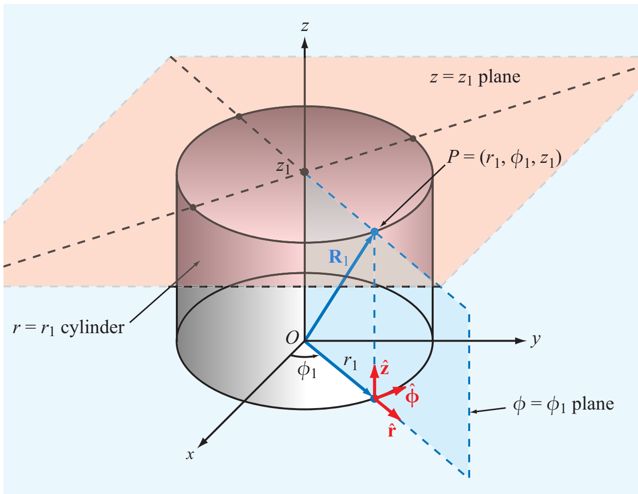

Fig. 2. Coordinate unit vectors in cylindrical. Coordinate system. Surfaces, in which one of the coordinates is constant, are also shown.

Gradient of a scalar field V in spherical coordinates:

$$\nabla V = \frac{\partial V}{\partial R}\hat{R} + \frac{1}{R}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}$$

For a vector field $\vec{A}$ in spherical coordinates, given in component form as: $\vec{A} = A_R\hat{R} + A_\theta\hat{\theta} + A_\phi\hat{\phi}$, we have the following expressions for the divergence and the curl:

$$\nabla \cdot \vec{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{R sin\theta}\left(\frac{\partial}{\partial\theta}(A_\phi sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right)\hat{R} + \frac{1}{R}\left(\frac{1}{sin\theta}\frac{\partial A_R}{\partial\phi} - \frac{\partial}{\partial R}(RA_\phi)\right)\hat{\theta} + \frac{1}{R}\left(\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial\theta}\right)\hat{\phi}$$

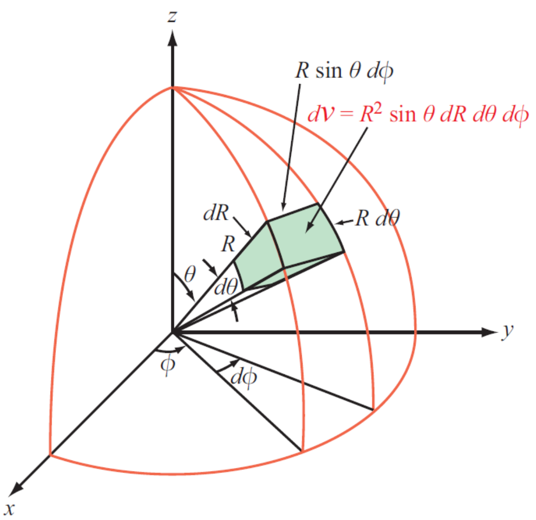

Fig. 1. Spherical coordinate system showing line and volume elements

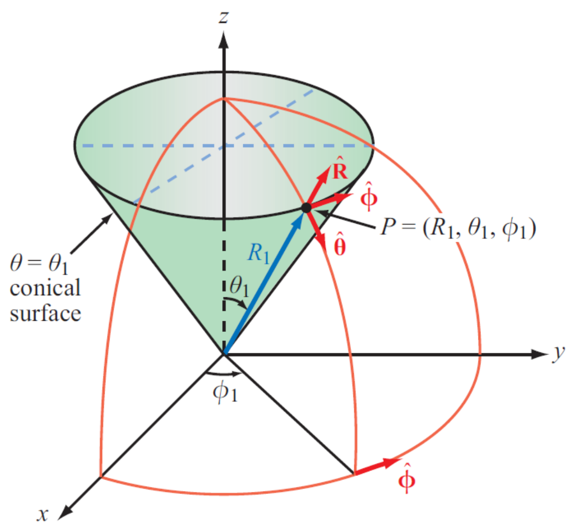

Fig. 2. Coordinate unit vectors in spherical coordinates

Table 3-1: Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\quad |\mathbf{A}| =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\quad \overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}} \cdot \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}} \cdot \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}} \cdot \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \cdot \hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}} \times \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\quad \mathbf{A} \cdot \mathbf{B} =$	$A_x B_x + A_y B_y + A_z B_z$	$A_r B_r + A_\phi B_\phi + A_z B_z$	$A_R B_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\quad \mathbf{A} \times \mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $\quad d\mathbf{l} =$	$\hat{\mathbf{x}}\, dx + \hat{\mathbf{y}}\, dy + \hat{\mathbf{z}}\, dz$	$\hat{\mathbf{r}}\, dr + \hat{\boldsymbol{\phi}} r\, d\phi + \hat{\mathbf{z}}\, dz$	$\hat{\mathbf{R}}\, dR + \hat{\boldsymbol{\theta}} R\, d\theta + \hat{\boldsymbol{\phi}} R \sin\theta\, d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\, dy\, dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\, dx\, dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\, dx\, dy$	$d\mathbf{s}_r = \hat{\mathbf{r}} r\, d\phi\, dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\, dr\, dz$ $d\mathbf{s}_z = \hat{\mathbf{z}} r\, dr\, d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}} R^2 \sin\theta\, d\theta\, d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}} R \sin\theta\, dR\, d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}} R\, dR\, d\theta$
Differential volume $\quad dV =$	$dx\, dy\, dz$	$r\, dr\, d\phi\, dz$	$R^2 \sin\theta\, dR\, d\theta\, d\phi$

Table 3-2: Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}} \cos\phi + \hat{\mathbf{y}} \sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}} \sin\phi + \hat{\mathbf{y}} \cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x \cos\phi + A_y \sin\phi$ $A_\phi = -A_x \sin\phi + A_y \cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}} \cos\phi - \hat{\boldsymbol{\phi}} \sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}} \sin\phi + \hat{\boldsymbol{\phi}} \cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r \cos\phi - A_\phi \sin\phi$ $A_y = A_r \sin\phi + A_\phi \cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}} \sin\theta \cos\phi + \hat{\mathbf{y}} \sin\theta \sin\phi + \hat{\mathbf{z}} \cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}} \cos\theta \cos\phi + \hat{\mathbf{y}} \cos\theta \sin\phi - \hat{\mathbf{z}} \sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}} \sin\phi + \hat{\mathbf{y}} \cos\phi$	$A_R = A_x \sin\theta \cos\phi + A_y \sin\theta \sin\phi + A_z \cos\theta$ $A_\theta = A_x \cos\theta \cos\phi + A_y \cos\theta \sin\phi - A_z \sin\theta$ $A_\phi = -A_x \sin\phi + A_y \cos\phi$
Spherical to Cartesian	$x = R \sin\theta \cos\phi$ $y = R \sin\theta \sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}} \sin\theta \cos\phi + \hat{\boldsymbol{\theta}} \cos\theta \cos\phi - \hat{\boldsymbol{\phi}} \sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}} \sin\theta \sin\phi + \hat{\boldsymbol{\theta}} \cos\theta \sin\phi + \hat{\boldsymbol{\phi}} \cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}} \cos\theta - \hat{\boldsymbol{\theta}} \sin\theta$	$A_x = A_R \sin\theta \cos\phi + A_\theta \cos\theta \cos\phi - A_\phi \sin\phi$ $A_y = A_R \sin\theta \sin\phi + A_\theta \cos\theta \sin\phi + A_\phi \cos\phi$ $A_z = A_R \cos\theta - A_\theta \sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}} \sin\theta + \hat{\mathbf{z}} \cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}} \cos\theta - \hat{\mathbf{z}} \sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r \sin\theta + A_z \cos\theta$ $A_\theta = A_r \cos\theta - A_z \sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}} \sin\theta + \hat{\boldsymbol{\theta}} \cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}} \cos\theta - \hat{\boldsymbol{\theta}} \sin\theta$	$A_r = A_R \sin\theta + A_\theta \cos\theta$ $A_\phi = A_\phi$ $A_z = A_R \cos\theta - A_\theta \sin\theta$

Divergence Theorem converts the integration over a volume to one over the surface enclosing that volume and vice versa.

$$\int_V \left(\nabla \cdot \vec{E}\right) dV = \oint_S \vec{E} \cdot \overrightarrow{dS}$$

Stokes's theorem converts the surface integral of the curl of a vector field on an open surface S, into a line integral of the vector field along the contour C bordering the surface S.

$$\int_S \left(\nabla \times \vec{B}\right) \cdot \overrightarrow{dS} = \oint_C \vec{B} \cdot \overrightarrow{dl}$$

Maxwell's Equations (point form)

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Maxwell's Equations (integral Form)

$$\oint_S \vec{D} \cdot \overrightarrow{dS} = Q$$

$$\oint_C \vec{E} \cdot \overrightarrow{dl} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \overrightarrow{dS}$$

$$\oint_S \vec{B} \cdot \overrightarrow{dS} = 0$$

$$\oint_C \vec{H} \cdot \overrightarrow{dl} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \cdot \overrightarrow{dS}$$

With

$$\vec{D} = \varepsilon \vec{E}$$
$$\vec{B} = \mu \vec{H}$$

In vacuum: $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$

Boundary conditions for $\vec{E}$ and $\vec{D}$ at the surface between two dielectrics:

$D_{1n} - D_{2n} = \rho_S$ where ρ_S is the free surface charge (excluding polarization charges)

$E_{1T} = E_{2T}$

Boundary conditions for $\vec{B}$ and $\vec{H}$ at the surface between two magnetic materials:

$H_{1T} - H_{2T} = J_S$ where J_S is the free surface current (excluding magnetizing currents)

$B_{1n} = B_{2n}$

The electrical potential difference between a and b is given by:

$V_{ab} = V_a - V_b = -\int_b^a \vec{E} \cdot \overrightarrow{dl}$

Resistance

$$R = \frac{V}{I} = \frac{-\int_l \vec{E} \cdot \overrightarrow{dl}}{\int_s \vec{J} \cdot \overrightarrow{dS}} = \frac{-\int_l \vec{E} \cdot \overrightarrow{dl}}{\int_s \sigma\vec{E} \cdot \overrightarrow{dS}}$$

Capacitance

$$C = \frac{Q}{V} = \frac{\int_s \vec{E} \cdot \overrightarrow{dS}}{-\int_l \vec{E} \cdot \overrightarrow{dl}}$$

Energy stored in a capacitor

$$W_e = \frac{1}{2} C V^2$$

Energy stored in inductor

$$W_m = \frac{1}{2} L I^2$$

Inductance L,

$$L = \frac{\Lambda}{I}$$

Where Λ is the magnetic flux linkage and *I* is the current.

Euler’s identity

$$e^{j\omega t} = cos(\omega t) + j\sin(\omega t)$$

A vector $\vec{\boldsymbol{A}}(z,t) = \vec{\boldsymbol{b}}(z)\cos(\omega t + \phi(z))$ can be written in phasor notation as:

$$\vec{\boldsymbol{A}}(z,t) = real\left(\tilde{\boldsymbol{b}}(\boldsymbol{z})\, e^{j\omega t}\right)$$

where $\tilde{\boldsymbol{b}}(\boldsymbol{z}) = \vec{\boldsymbol{b}}(\boldsymbol{z})e^{j\phi(z)}$ is the phasor, $\phi(z)$ is a function of *z* and $\vec{\boldsymbol{b}}(\boldsymbol{z})$ a real vector.

Maxwell’s equations for phasors

$$\nabla \times \tilde{H} = j\omega\varepsilon\tilde{E}$$

$$\nabla \times \tilde{E} = -j\omega\mu\tilde{H}$$

Speed of light as a function of permeability μ_0 and permittivity ε_0 of the vacuum

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

Derivatives and Integrals (all the integrals are defined up to an additive constant)

$$\frac{d}{dx}x^n = nx^{(n-1)}$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$$

$$\int \sin(x)\, dx = -\cos(x)$$

$$\int \cos(x) dx = \sin(x)$$

$$\int \cos(x)\sin(x)\, dx = \frac{(\sin(x))^2}{2}$$

$$\int \frac{1}{x} dx = \ln(x)$$

Area and volume of sphere

$$A = 4\pi r^2 \qquad V = \frac{4\pi r^3}{3}$$

Sine and cosine table angles

x	0	π/6	π/4	π/3	π/2	π	3π/2	2π
sin(x)	0	1/2	1/ √2	√3 / 2	1	0	-1	0
cos(x)	1	√3 / 2	1/ √2	1/2	0	-1	0	1

$$\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$$