

(*van der Waals EOS in term of reduced variables*)

$$p \rightarrow \frac{3 - 9v + 8v^2 \tau}{v^2 (-1 + 3v)}$$

(*enthalpy in term of reduced variables*)

$$h \rightarrow \frac{4\tau(5v-1)}{3v-1} - \frac{6}{v}$$

(*One can solve enthalpy equation for V(h) roots*)

$$\text{In[2]:= Solve}\left[\frac{4\tau(5v-1)}{3v-1} - \frac{6}{v} - h == 0, v\right]$$

$$\text{Out[2]= } \left\{ \left\{ v \rightarrow \frac{-18 + h - 4\tau + \sqrt{324 + 36h + h^2 - 336\tau - 8h\tau + 16\tau^2}}{2(3h - 20\tau)} \right\}, \right. \\ \left. \left\{ v \rightarrow \frac{18 - h + 4\tau + \sqrt{324 + 36h + h^2 - 336\tau - 8h\tau + 16\tau^2}}{2(-3h + 20\tau)} \right\} \right\}$$

$$\text{In[5]:= FullSimplify}\left[\frac{18 - h + 4\tau + \sqrt{324 + 36h + h^2 - 336\tau - 8h\tau + 16\tau^2}}{2(-3h + 20\tau)}\right]$$

$$\text{Out[5]= } \frac{-18 + h - 4\tau - \sqrt{(18 + h)^2 - 8(42 + h)\tau + 16\tau^2}}{6h - 40\tau}$$

(*Here we have two solutions and these root give the same result. Substituting one of these roots into EOS state,*)

$$\text{In[6]:= Limit}\left[\frac{3 - 9v + 8v^2 \tau}{v^2 (-1 + 3v)}, v \rightarrow \frac{-18 + h - 4\tau - \sqrt{(18 + h)^2 - 8(42 + h)\tau + 16\tau^2}}{6h - 40\tau}\right]$$

$$\text{Out[6]= } - \left(\left(4(3h - 20\tau) \left(h^2(27 + 8\tau) + h(-64\tau^2 + 27(18 + \sqrt{324 + 36h + h^2 - 336\tau - 8h\tau + 16\tau^2})) - \right. \right. \right. \\ \left. \left. 8\tau(54 + \sqrt{324 + 36h + h^2 - 336\tau - 8h\tau + 16\tau^2}) \right) + \right. \\ \left. 4\tau(32\tau^2 - 9(18 + \sqrt{324 + 36h + h^2 - 336\tau - 8h\tau + 16\tau^2})) + \right. \\ \left. 4\tau(57 + 2\sqrt{324 + 36h + h^2 - 336\tau - 8h\tau + 16\tau^2}) \right) \Big) \Big) / \\ \left(\left(18 - h + 4\tau + \sqrt{324 + 36h + h^2 - 336\tau - 8h\tau + 16\tau^2} \right)^2 \right. \\ \left. \left(54 + 3h - 28\tau + 3\sqrt{324 + 36h + h^2 - 336\tau - 8h\tau + 16\tau^2} \right) \right) \Big)$$

$$\text{In[7]:= FullSimplify[%]}$$

$$\text{Out[7]= } \frac{1}{24} \left(-h^2 + h \left(36 + 8\tau - \sqrt{(18 + h)^2 - 8(42 + h)\tau + 16\tau^2} \right) + \right. \\ \left. 2 \left(-9 \left(-18 + \sqrt{(18 + h)^2 - 8(42 + h)\tau + 16\tau^2} \right) + \right. \right. \\ \left. \left. 2\tau \left(-60 - 4\tau + \sqrt{(18 + h)^2 - 8(42 + h)\tau + 16\tau^2} \right) \right) \right)$$

(*And solving for \tau*)

$$\left(\left(21\,749\,760\,p^2 - 49\,152\,h\,p^2 - 8192\,p^3 \right)^2 \right)^{1/3} - \frac{1}{1920 \times 2^{1/3}} \\
\left(1 + i\sqrt{3} \right) \left(27\,648\,000 - 77\,414\,400\,h - 22\,855\,680\,h^2 - 65\,536\,h^3 + 724\,377\,600\,p - \right. \\
38\,338\,560\,h\,p - 98\,304\,h^2\,p - 21\,749\,760\,p^2 - 49\,152\,h\,p^2 - 8192\,p^3 + \\
\left. \sqrt{4 \left(-57\,600 + 107\,520\,h - 1024\,h^2 + 238\,080\,p - 1024\,h\,p - 256\,p^2 \right)^3 + \right. \\
\left. \left(27\,648\,000 - 77\,414\,400\,h - 22\,855\,680\,h^2 - 65\,536\,h^3 + 724\,377\,600\,p - \right. \right. \\
\left. \left. 38\,338\,560\,h\,p - 98\,304\,h^2\,p - 21\,749\,760\,p^2 - 49\,152\,h\,p^2 - 8192\,p^3 \right)^2 \right)^{1/3} \Bigg\} \Bigg\}$$

(*We obtain a tree solutions. One of these solution is reel.*)

(*h=1,2,3*)

$$\text{In[9]:= Limit} \left[\frac{1}{60} \left(15 + 13\,h - p \right) - \left(-57\,600 + 107\,520\,h - 1024\,h^2 + 238\,080\,p - 1024\,h\,p - 256\,p^2 \right) \right] / \\
\left(480 \times 2^{2/3} \left(27\,648\,000 - 77\,414\,400\,h - 22\,855\,680\,h^2 - 65\,536\,h^3 + 724\,377\,600\,p - \right. \right. \\
38\,338\,560\,h\,p - 98\,304\,h^2\,p - 21\,749\,760\,p^2 - 49\,152\,h\,p^2 - 8192\,p^3 + \\
\left. \sqrt{4 \left(-57\,600 + 107\,520\,h - 1024\,h^2 + 238\,080\,p - 1024\,h\,p - 256\,p^2 \right)^3 + \right. \\
\left. \left(27\,648\,000 - 77\,414\,400\,h - 22\,855\,680\,h^2 - 65\,536\,h^3 + 724\,377\,600\,p - \right. \right. \\
\left. \left. 38\,338\,560\,h\,p - 98\,304\,h^2\,p - 21\,749\,760\,p^2 - 49\,152\,h\,p^2 - 8192\,p^3 \right)^2 \right)^{1/3} \Bigg) + \\
\frac{1}{960 \times 2^{1/3}} \left(27\,648\,000 - 77\,414\,400\,h - 22\,855\,680\,h^2 - 65\,536\,h^3 + 724\,377\,600\,p - \right. \\
38\,338\,560\,h\,p - 98\,304\,h^2\,p - 21\,749\,760\,p^2 - 49\,152\,h\,p^2 - 8192\,p^3 + \\
\left. \sqrt{4 \left(-57\,600 + 107\,520\,h - 1024\,h^2 + 238\,080\,p - 1024\,h\,p - 256\,p^2 \right)^3 + \right. \\
\left. \left(27\,648\,000 - 77\,414\,400\,h - 22\,855\,680\,h^2 - 65\,536\,h^3 + 724\,377\,600\,p - 38\,338\,560 \right. \right. \\
\left. \left. h\,p - 98\,304\,h^2\,p - 21\,749\,760\,p^2 - 49\,152\,h\,p^2 - 8192\,p^3 \right)^2 \right)^{1/3}, h \rightarrow 1 \Bigg]$$

$$\text{Out[9]=} \frac{1}{60} \left(28 - p + \left(-191 - 926\,p + p^2 \right) \right) / \\
\left(-8873 + 83\,733\,p - 2661\,p^2 - p^3 + 90 \sqrt{(46 + p)^2 (5 - 81\,p + 444\,p^2 + p^3)} \right)^{1/3} + \\
\left(-8873 + 83\,733\,p - 2661\,p^2 - p^3 + 90 \sqrt{(46 + p)^2 (5 - 81\,p + 444\,p^2 + p^3)} \right)^{1/3}$$

$$\begin{aligned} \text{In[10]:= } & \text{Limit}\left[\frac{1}{60} (15 + 13 h - p) - (-57600 + 107520 h - 1024 h^2 + 238080 p - 1024 h p - 256 p^2) \right. \\ & \left. \left(480 \times 2^{2/3} \left(27648000 - 77414400 h - 22855680 h^2 - 65536 h^3 + 724377600 p - \right. \right. \right. \\ & \quad 38338560 h p - 98304 h^2 p - 21749760 p^2 - 49152 h p^2 - 8192 p^3 + \\ & \quad \left. \sqrt{4 (-57600 + 107520 h - 1024 h^2 + 238080 p - 1024 h p - 256 p^2)^3 +} \right. \\ & \quad \left. \left(27648000 - 77414400 h - 22855680 h^2 - 65536 h^3 + 724377600 p - \right. \right. \\ & \quad \left. \left. 38338560 h p - 98304 h^2 p - 21749760 p^2 - 49152 h p^2 - 8192 p^3\right)^2\right)^{1/3}\right) + \\ & \frac{1}{960 \times 2^{1/3}} \left(27648000 - 77414400 h - 22855680 h^2 - 65536 h^3 + 724377600 p - \right. \\ & \quad 38338560 h p - 98304 h^2 p - 21749760 p^2 - 49152 h p^2 - 8192 p^3 + \\ & \quad \left. \sqrt{4 (-57600 + 107520 h - 1024 h^2 + 238080 p - 1024 h p - 256 p^2)^3 +} \right. \\ & \quad \left. \left(27648000 - 77414400 h - 22855680 h^2 - 65536 h^3 + 724377600 p - 38338560 \right. \right. \\ & \quad \left. \left. h p - 98304 h^2 p - 21749760 p^2 - 49152 h p^2 - 8192 p^3\right)^2\right)^{1/3}, h \rightarrow 2] \end{aligned}$$

$$\begin{aligned} \text{Out[10]= } & \frac{1}{60} \left(41 - p + (-599 - 922 p + p^2) \right. \\ & \left. \left(-26749 + 79017 p - 2667 p^2 - p^3 + 90 \sqrt{(47 + p)^2 (52 - 183 p + 450 p^2 + p^3)}\right)^{1/3} + \right. \\ & \left. \left(-26749 + 79017 p - 2667 p^2 - p^3 + 90 \sqrt{(47 + p)^2 (52 - 183 p + 450 p^2 + p^3)}\right)^{1/3}\right) \end{aligned}$$

$$\begin{aligned} \text{In[11]:= } & \text{Limit}\left[\frac{1}{60} (15 + 13 h - p) - (-57600 + 107520 h - 1024 h^2 + 238080 p - 1024 h p - 256 p^2) \right. \\ & \left. \left(480 \times 2^{2/3} \left(27648000 - 77414400 h - 22855680 h^2 - 65536 h^3 + 724377600 p - \right. \right. \right. \\ & \quad 38338560 h p - 98304 h^2 p - 21749760 p^2 - 49152 h p^2 - 8192 p^3 + \\ & \quad \left. \sqrt{4 (-57600 + 107520 h - 1024 h^2 + 238080 p - 1024 h p - 256 p^2)^3 +} \right. \\ & \quad \left. \left(27648000 - 77414400 h - 22855680 h^2 - 65536 h^3 + 724377600 p - \right. \right. \\ & \quad \left. \left. 38338560 h p - 98304 h^2 p - 21749760 p^2 - 49152 h p^2 - 8192 p^3\right)^2\right)^{1/3}\right) + \\ & \frac{1}{960 \times 2^{1/3}} \left(27648000 - 77414400 h - 22855680 h^2 - 65536 h^3 + 724377600 p - \right. \\ & \quad 38338560 h p - 98304 h^2 p - 21749760 p^2 - 49152 h p^2 - 8192 p^3 + \\ & \quad \left. \sqrt{4 (-57600 + 107520 h - 1024 h^2 + 238080 p - 1024 h p - 256 p^2)^3 +} \right. \\ & \quad \left. \left(27648000 - 77414400 h - 22855680 h^2 - 65536 h^3 + 724377600 p - 38338560 \right. \right. \\ & \quad \left. \left. h p - 98304 h^2 p - 21749760 p^2 - 49152 h p^2 - 8192 p^3\right)^2\right)^{1/3}, h \rightarrow 3] \end{aligned}$$

$$\begin{aligned} \text{Out[11]= } & \frac{1}{60} \left(54 - p + (-999 - 918 p + p^2) \right. \\ & \left. \left(-50301 + 74277 p - 2673 p^2 - p^3 + 90 \sqrt{(48 + p)^2 (189 - 261 p + 456 p^2 + p^3)}\right)^{1/3} + \right. \\ & \left. \left(-50301 + 74277 p - 2673 p^2 - p^3 + 90 \sqrt{(48 + p)^2 (189 - 261 p + 456 p^2 + p^3)}\right)^{1/3}\right) \end{aligned}$$

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In[18]:= Plot[ {  $\frac{1}{60} \left( 28 - p + (-191 - 926 p + p^2) \right) /$   

 $\left( -8873 + 83733 p - 2661 p^2 - p^3 + 90 \sqrt{(46 + p)^2 (5 - 81 p + 444 p^2 + p^3)} \right)^{1/3} +$   

 $\left( -8873 + 83733 p - 2661 p^2 - p^3 + 90 \sqrt{(46 + p)^2 (5 - 81 p + 444 p^2 + p^3)} \right)^{1/3} \right),$   

 $\frac{1}{60} \left( 41 - p + (-599 - 922 p + p^2) \right) /$   

 $\left( -26749 + 79017 p - 2667 p^2 - p^3 + 90 \sqrt{(47 + p)^2 (52 - 183 p + 450 p^2 + p^3)} \right)^{1/3} +$   

 $\left( -26749 + 79017 p - 2667 p^2 - p^3 + 90 \sqrt{(47 + p)^2 (52 - 183 p + 450 p^2 + p^3)} \right)^{1/3} \right),$   

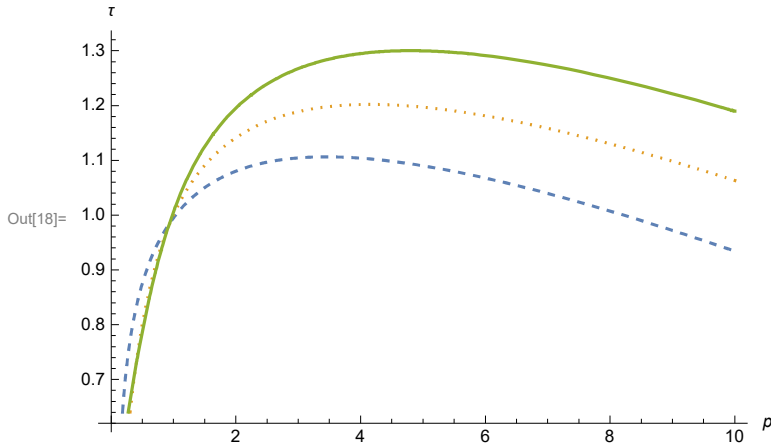
 $\frac{1}{60} \left( 54 - p + (-999 - 918 p + p^2) \right) /$   

 $\left( -50301 + 74277 p - 2673 p^2 - p^3 + 90 \sqrt{(48 + p)^2 (189 - 261 p + 456 p^2 + p^3)} \right)^{1/3} +$   

 $\left( -50301 + 74277 p - 2673 p^2 - p^3 + 90 \sqrt{(48 + p)^2 (189 - 261 p + 456 p^2 + p^3)} \right)^{1/3} \right) \},$   

{p, 0, 10}, PlotStyle -> {Dashed, Dotted, Solid},  

AxesLabel -> {p,  $\tau$ } ]
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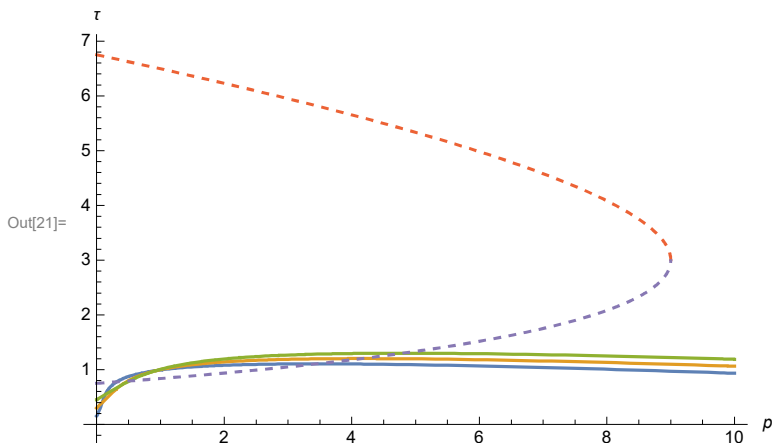


(*Here dashe, dotted and solid lines correspond to h=1,2,3 respectively*)
(*Upper and lower inversion curves*)

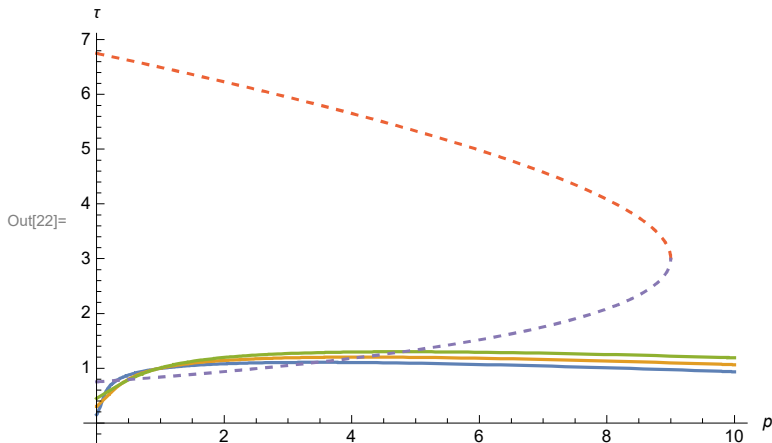
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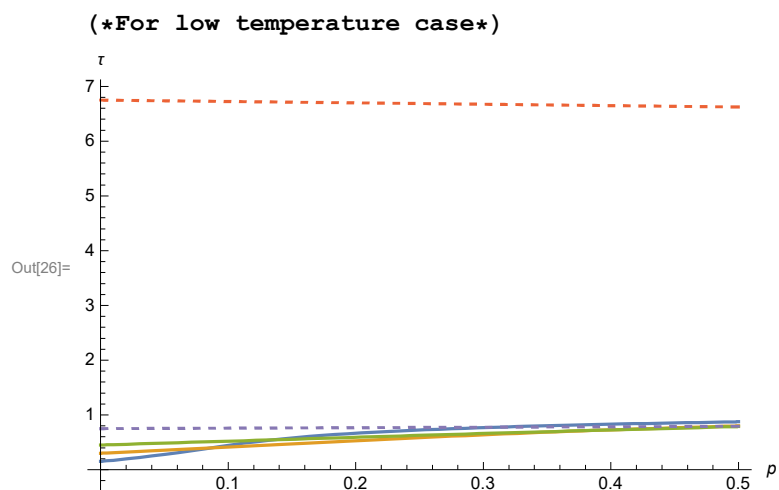
In[21]:= Plot[ { 1/60 (28 - p + (-191 - 926 p + p^2) /
  (-8873 + 83 733 p - 2661 p^2 - p^3 + 90 sqrt((46 + p)^2 (5 - 81 p + 444 p^2 + p^3)))^(1/3) +
  (-8873 + 83 733 p - 2661 p^2 - p^3 + 90 sqrt((46 + p)^2 (5 - 81 p + 444 p^2 + p^3)))^(1/3) ),
  1/60 (41 - p + (-599 - 922 p + p^2) /
  (-26 749 + 79 017 p - 2667 p^2 - p^3 + 90 sqrt((47 + p)^2 (52 - 183 p + 450 p^2 + p^3)))^(1/3) +
  (-26 749 + 79 017 p - 2667 p^2 - p^3 + 90 sqrt((47 + p)^2 (52 - 183 p + 450 p^2 + p^3)))^(1/3) ),
  1/60 (54 - p + (-999 - 918 p + p^2) /
  (-50 301 + 74 277 p - 2673 p^2 - p^3 + 90 sqrt((48 + p)^2 (189 - 261 p + 456 p^2 + p^3)))^(1/3) +
  (-50 301 + 74 277 p - 2673 p^2 - p^3 + 90 sqrt((48 + p)^2 (189 - 261 p + 456 p^2 + p^3)))^(1/3) ),
  3 + (9 - p)/12 + sqrt(9 - p), 3 + (9 - p)/12 - sqrt(9 - p) }, {p, 0, 10},
  AxesLabel -> {p, tau}, PlotStyle ->
  {Solid, Solid, Solid, Dashed, Dashed}]

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(*Combining two graphs*)





(*It seems isenthalpic curves has positive
slope below the lower inversion cure(dashed blue line)*)