# Matrix sieve - new algorithm for finding primes

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Abstract

A new sieving algorithm based on derived matrix expression of prime numbers is proposed. The algorithm allows to calculate indexes $p$ of prime numbers in two sequences: $S_1(p) = 5 + 6p = 5,11,17, \ldots;\ p = 0,1,2,3, \ldots$ and $S_2(p) = 7 + 6p = 7,13,19, \ldots;\ p = 0,1,2,3, \ldots$ in a given range of natural numbers $(N_1, N_2)$. Also general primality criteria is formulated. C++ program for finding primes in any given range $(N_1, N_2)$ and C++ program for primality testing of given natural number N are presented in Attachments 1 and 2.

## 1. *Matrix general expression for composite numbers*

Consider sequence of natural numbers from which members divisible by 2 and 3 are removed

$$S(p) = 5,7,11,13,17,19,23,25,29, \ldots = \begin{cases} 3p + 5, p = 0,2,4,6,8, \ldots \\ 3(p-1) + 7, p = 1,3,5,7, \ldots \end{cases} \qquad (1)$$

Sequence $S(p)$ can be divided into two sequences $S_1(i)$ and $S_2(i)$:

$$S_1(i) = 5 + 6i = 5,11,17, \ldots;\ i = 0,1,2,3, \ldots \qquad (2)$$

$$S_2(i) = 7 + 6i = 7,13,19, \ldots;\ i = 0,1,2,3, \ldots \qquad (3)$$

Sequence $S(p)$ contains all primes (except 2 and 3) and, using definitions of $S_1(i)$ and $S_2(i)$, all composite numbers (except divisible by 2 and 3) of four different types:

$$FF(i,j) = S_1(i) * S_1(j) = (5+6i)(5+6j); \qquad SS(i,j) = S_2(i) * S_2(j) = (7+6i)(7+6j);$$

$$SF(i,j) = S_1(i) * S_2(j) = (5+6i)(7+6j); \qquad FS(i,j) = S_1(j) * S_2(i) = (5+6j)(7+6i);$$

which constitute four 2-dimensional arrays:

$$FF(i,j) = 25 + 30(i+j) + 36ij = \begin{pmatrix} 25 & 55 & 85 & 115 & 145 & \ldots \\ 55 & 121 & 187 & 253 & 319 & \ldots \\ 85 & 187 & 289 & 391 & 493 & \ldots \\ 115 & 253 & 391 & 529 & 667 & \ldots \\ 145 & 319 & 493 & 667 & 841 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}; i,j = 0,1,2,3,4 \ldots. \qquad (4)$$

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$$SS(i,j) = 49 + 42(i+j) + 36ij = \begin{pmatrix} 49 & 91 & 133 & 175 & 217 & \dots \\ 91 & 169 & 247 & 325 & 403 & \dots \\ 133 & 247 & 361 & 475 & 589 & \dots \\ 175 & 325 & 475 & 625 & 775 & \dots \\ 217 & 403 & 589 & 775 & 961 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix};\ i,j = 0,1,2,3,4\ \dots. \tag{5}$$

$$FS(i,j) = 35 + 6(5i+7j) + 36ij = \begin{pmatrix} 35 & 77 & 119 & 161 & 203 & \dots \\ 65 & 143 & 221 & 299 & 377 & \dots \\ 95 & 209 & 323 & 437 & 551 & \dots \\ 125 & 275 & 425 & 575 & 725 & \dots \\ 155 & 341 & 527 & 713 & 899 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix};\ i,j = 0,1,2,3,4\ \dots. \tag{6}$$

.

$$SF(i,j) = 35 + 6(7i+5j) + 36ij = \begin{pmatrix} 35 & 65 & 95 & 125 & 155 & \dots \\ 77 & 143 & 209 & 275 & 341 & \dots \\ 119 & 221 & 323 & 425 & 527 & \dots \\ 161 & 299 & 437 & 575 & 713 & \dots \\ 203 & 377 & 551 & 725 & 899 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix};\ i,j = 0,1,2,3,4\ \dots. \tag{7}$$

.

Substituting numbers in the above matrices with their corresponding indexes $p$ in accordance with equation (1), the following four matrices can be obtained:

$$ff(i,j) = \begin{pmatrix} 7 & 17 & 27 & 37 & \dots & & \dots \\ 17 & 39 & 61 & 83 & \dots & & \dots \\ 27 & 61 & 95 & 129 & \dots & & \dots \\ 37 & 83 & 129 & 175 & \dots & & \dots \\ 47 & 105 & 163 & 221 & \dots & & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \dots \end{pmatrix} \tag{8}$$

$$ss(i,j) = \begin{pmatrix} 15 & 29 & 43 & 57 & \dots & & \dots \\ 29 & 55 & 81 & 107 & \dots & & \dots \\ 43 & 81 & 119 & 157 & \dots & & \dots \\ 57 & 107 & 157 & 207 & \dots & & \dots \\ 71 & 133 & 195 & 257 & \dots & & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \dots \end{pmatrix} \tag{9}$$

$$fs(i,j) = \begin{pmatrix} 10 & 24 & 38 & 52 & \dots & & \dots \\ 20 & 46 & 72 & 98 & \dots & & \dots \\ 30 & 68 & 106 & 144 & \dots & & \dots \\ 40 & 90 & 140 & 190 & \dots & & \dots \\ 50 & 112 & 174 & 236 & \dots & & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \dots \end{pmatrix} \tag{10}$$

$$sf(i,j) = \begin{pmatrix} 10 & 20 & 30 & 40 & \dots & & \dots \\ 24 & 46 & 68 & 90 & \dots & & \dots \\ 38 & 72 & 106 & 140 & \dots & & \dots \\ 52 & 98 & 144 & 190 & \dots & & \dots \\ 66 & 124 & 182 & 240 & \dots & & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \dots \end{pmatrix} \tag{11}$$

In general form these arrays can be expressed as

$$ff(i,j) = 12ij - 2(i+j) - 1; i,j = 1,2,3,4 \ldots. \quad (12)$$

$$ss(i,j) = 12ij + 2(i+j) - 1;\ i,j = 1,2,3,4 \ldots. \quad (13)$$

$$fs(i,j) = 12ij - 2(i-j) - 2; i,j = 1,2,3,4 \ldots \quad (14)$$

$$sf(i,j) = 12ij + 2(i-j) - 2;\ i,j = 1,2,3,4 \ldots. \quad (15)$$

## 2. *Matrix general expression of prime numbers*

Note that arrays $ff(i,j)$ and $ss(i,j)$ are symmetric and are comprised of odd integers (i.e. odd indexes of elements of $S(p)$); arrays $sf(i,j)$ and $fs(i,j)$ are transposes of each other and are comprised of even integers (i.e. even indexes of elements of $S(p)$). Thus, all elements of arrays $ff(i,j)$ and $ss(i,j)$ are indexes of members of $S_2\ (p)$ and all elements of arrays $sf\ \ (i,j)$ and $fs(i,j)$ are indexes of members of $S_1(p)$. Let us now consider sequences $S_1(P)$ and $S_2(P)$ separately as defined below

$$S_1(P) = 6P + 5; \quad P = 0,1,2,3, \ldots = p/2 \quad (16)$$

$$S_2(P) = 6P + 7; \quad P = 0,1,2,3, \ldots = (p-1)/2 \quad (17)$$

Indexes $P_1(i,j)$, $P_2(i,j)$ and $P_3(i,j)$, $P_4(i,j)$ corresponding to composite numbers in $S_1(P)$ and $S_2(P)$ respectively are then defined as:

for $S_1(P)$

$$P_1(i,j) = 6ij - i + j - 1 = \begin{pmatrix} 5 & 12 & 19 & 26 & 33 & 40 & \\ 0 & 23 & 36 & 49 & 62 & 75 & \ldots \\ 0 & 0 & 53 & 72 & 91 & 110 & \ldots \\ 0 & 0 & 0 & 95 & 120 & 145 & \ldots \\ 0 & 0 & 0 & 0 & 149 & 180 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ldots & 215 & \ldots \end{pmatrix};\ i = 1,2,3,4 \ldots;\ j \geq i \quad (18)$$

$$P_2(i,j) = 6ij + i - j - 1 = \begin{pmatrix} 0 & 10 & 15 & 20 & 25 & 30 & \ldots \\ 0 & 0 & 34 & 45 & 56 & 67 & \ldots \\ 0 & 0 & 0 & 70 & 87 & 104 & \ldots \\ 0 & 0 & 0 & 0 & 118 & 141 & \ldots \\ 0 & 0 & 0 & 0 & 0 & 178 & \ldots \\ . & . & . & . & . & 0 & \ldots \end{pmatrix}; i = 1,2,3,4 \ldots;\ j \geq i + 1 \quad (19)$$

for $S_2(P)$

$$P_3(i,j) = 6ij - i - j - 1 = \begin{pmatrix} 3 & 8 & 13 & 18 & 23 & 28 & \ldots \\ 0 & 19 & 30 & 41 & 52 & 63 & \ldots \\ 0 & 0 & 47 & 64 & 81 & 98 & \ldots \\ 0 & 0 & 0 & 87 & 110 & 133 & \ldots \\ 0 & 0 & 0 & 0 & 139 & 168 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ldots & 203 & \ldots \end{pmatrix}\ ;\ i = 1,2,3,4 \ldots;\ j \geq i \quad (20)$$

$$P_4(i,j) = 6ij + i + j - 1 = \begin{pmatrix} 7 & 14 & 21 & 28 & 35 & 42 & \dots \\ 0 & 27 & 40 & 53 & 66 & 79 & \dots \\ 0 & 0 & 59 & 78 & 97 & 116 & \dots \\ 0 & 0 & 0 & 103 & 128 & 153 & \dots \\ 0 & 0 & 0 & 0 & 159 & 190 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & 227 & \dots \end{pmatrix} \quad ; \quad i = 1,2,3,4 \dots ;\ j \geq i \qquad (21)$$

Sequence $S_1(P)$ contains members of two arrays of composite numbers, indexes of which are contained in arrays (18), (19) and subsequence of prime numbers Primes-in-S1.

Sequence $S_2(P)$ contains members of two arrays of composite numbers, indexes of which are contained in arrays (20), (21) and subsequence of prime numbers Primes-in-S2.

So, in order to find all prime numbers – Primes-in-S1 and Primes-in-S2 - it is necessarily to exclude from sequence $S_1(P)$ members, indexes of which are contained in arrays (18), (19), and exclude from sequence $S_2(P)$ members, indexes of which are contained in arrays (20), (21) and matrix general expression of prime numbers can be formulated as follows:

Natural numbers that are not contained in arrays

$$6ij - i + j - 1 = \begin{pmatrix} 5 & 12 & 19 & 26 & 33 & 40 & \\ 0 & 23 & 36 & 49 & 62 & 75 & \dots \\ 0 & 0 & 53 & 72 & 91 & 110 & \dots \\ 0 & 0 & 0 & 95 & 120 & 145 & \dots \\ 0 & 0 & 0 & 0 & 149 & 180 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & 215 & \dots \end{pmatrix} ;\ i = 1,2,3 \dots ;\ j \geq i$$

$$6ij + i - j - 1 = \begin{pmatrix} 0 & 10 & 15 & 20 & 25 & 30 & \dots \\ 0 & 0 & 34 & 45 & 56 & 67 & \dots \\ 0 & 0 & 0 & 70 & 87 & 104 & \dots \\ 0 & 0 & 0 & 0 & 118 & 141 & \dots \\ 0 & 0 & 0 & 0 & 0 & 178 & \dots \\ . & . & . & . & . & 0 & \dots \end{pmatrix} ;\ i = 1,2,3, \dots ;\ j \geq i + 1$$

are indexes $P$ of all primes in the sequence $S_1(P) = 6P + 5;\ P = 0,1,2,3,\dots$

Natural numbers that are not contained in arrays

$$6ij - i - j - 1 = \begin{pmatrix} 3 & 8 & 13 & 18 & 23 & 28 & \dots \\ 0 & 19 & 30 & 41 & 52 & 63 & \dots \\ 0 & 0 & 47 & 64 & 81 & 98 & \dots \\ 0 & 0 & 0 & 87 & 110 & 133 & \dots \\ 0 & 0 & 0 & 0 & 139 & 168 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & 203 & \dots \end{pmatrix} ;\ i = 1,2; 3, \dots ;\ j \geq i$$

$$6ij + i + j - 1 = \begin{pmatrix} 7 & 14 & 21 & 28 & 35 & 42 & \dots \\ 0 & 27 & 40 & 53 & 66 & 79 & \dots \\ 0 & 0 & 59 & 78 & 97 & 116 & \dots \\ 0 & 0 & 0 & 103 & 128 & 153 & \dots \\ 0 & 0 & 0 & 0 & 159 & 190 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & 227 & \dots \end{pmatrix} ;\ i = 1,2,3, \dots ;\ j \geq i$$

are indexes $P$ of all primes in the sequence $S_2(P) = 6P + 7;\ P = 0,1,2,3,\dots$

Equations (18) – (21) provide algebraic relationships connecting integers $i$ and $j$ with indexes of composite numbers in $S_1(P)$ and $S_2(P)$. Now, we need to establish a relationship between our range of interest ($N_1$,$N_2$) and the corresponding area of change of integers $i$ and $j$:

Since $j \geq i$ , maximum value of $i$ will be when $i = j$ and it approximately equals

$$i\,max = \frac{\sqrt{N_2}}{6} \tag{22}$$

For array $P_1(i,j)$ we have: $jmin = \frac{\frac{N_1}{6}+\mathrm{i}+1}{6\mathrm{i}+1}$ ; $jmax = \frac{\frac{N_2}{6}+\mathrm{i}+1}{6\mathrm{i}+1}$; (23)

For array $P_2(i,j)$: $jmin = \frac{\frac{N_1}{6}-\mathrm{i}+1}{6\mathrm{i}-1}$ ; $jmax = \frac{\frac{N_2}{6}-\mathrm{i}+1}{6\mathrm{i}-1}$; (24)

For array $P_3(i,j)$: $jmin = \frac{\frac{N_1}{6}+\mathrm{i}+1}{6\mathrm{i}-1}$ ; $jmax = \frac{\frac{N_2}{6}+\mathrm{i}+1}{6\mathrm{i}-1}$; (25)

For array $P_2(i,j)$: $jmin = \frac{\frac{N_1}{6}-\mathrm{i}+1}{6\mathrm{i}+1}$ ; $jmax = \frac{\frac{N_2}{6}-\mathrm{i}+1}{6\mathrm{i}+1}$; (26)

Matrix sieving algorithm can be formulated as follows:

In order to find all prime numbers in the range from $N_1$ to $N_2$ it is necessarily to remove from sequence $S_1(P)$ members with indexes $P_1(i,j)$, $P_2(i,j)$:

$$S_1(P) = 6P + 5; \;\; P = 0,1,2,3, \ldots$$

$$P_1(i,j) = 6ij - i + j - 1; \;\; i = 1,2, \ldots, \left\lceil \frac{\sqrt{N_2}}{6} \right\rceil; \; j = \left\lceil \frac{\frac{N_1}{6}+\mathrm{i}+1}{6\mathrm{i}+1} \right\rceil, \left\lceil \frac{\frac{N_1}{6}+\mathrm{i}+1}{6\mathrm{i}+1} \right\rceil + 1, \ldots, \left\lceil \frac{\frac{N_2}{6}+i+1}{6i+1} \right\rceil; j \geq i$$

$$P_2(i,j) = 6ij + i - j - 1; \;\; i = 1,2, \ldots, \left\lceil \frac{\sqrt{N_2}}{6} \right\rceil; \;\; j = \left\lceil \frac{\frac{N_1}{6}-\mathrm{i}+1}{6\mathrm{i}-1} \right\rceil, \left\lceil \frac{\frac{N_1}{6}-\mathrm{i}+1}{6\mathrm{i}-1} \right\rceil + 1, \ldots, \left\lceil \frac{\frac{N_2}{6}-i+1}{6i-1} \right\rceil; \; j \geq i+1$$

And remove from sequence $S_2(P)$ members with indexes $P_3(i,j)$, $P_4(i,j)$:

$$S_2(P) = 6P + 7; \;\; P = 0,1,2,3, \ldots$$

$$P_3(i,j) = 6ij - i - j - 1; \;\; i = 1,2, \ldots, \left\lceil \frac{\sqrt{N_2}}{6} \right\rceil; \; j = \left\lceil \frac{\frac{N_1}{6}+\mathrm{i}+1}{6\mathrm{i}-1} \right\rceil, \left\lceil \frac{\frac{N_1}{6}+\mathrm{i}+1}{6\mathrm{i}-1} \right\rceil + 1, \ldots, \left\lceil \frac{\frac{N_2}{6}+i+1}{6i-1} \right\rceil; j \geq i$$

$$P_4(i,j) = 6ij + i + j - 1; \;\; i = 1,2, \ldots, \left\lceil \frac{\sqrt{N_2}}{6} \right\rceil; \;\; j = \left\lceil \frac{\frac{N_1}{6}-\mathrm{i}+1}{6\mathrm{i}+1} \right\rceil, \left\lceil \frac{\frac{N_1}{6}-\mathrm{i}+1}{6\mathrm{i}+1} \right\rceil + 1, \ldots, \left\lceil \frac{\frac{N_2}{6}-i+1}{6i+1} \right\rceil; \; j \geq i$$

This algorithm is deterministic and allows finding primes using only simple calculations. For example, let us find primes in the range, say, $N_1$ =100, $N_2$ =300.

Calculate

$P_{\min}$=100/6=16; $P_{\max}$=300/6=50.

Pick up members of arrays $P_1(i,j)$, $P_2(i,j)$ in the range (16;50) we have;

19,23,26,33,36,40,47,49, - from $P_1(i,j)$,

20,25,30,34.35,40,45,50 - from $P_2(i,j)$.

Indexes $P$ of members of the sequence $S_1(P) = 6P + 5$ that do not appear in these arrays are:

16,17,18,21,22,24,27,28,29,31,32,37,38,39,41,42,43,44,46,48

And Primes-in-S1 are:

101,107,113,131,137,149,167,173,179,191,197,227,233,239,251,257,263,269,281,293

Pick up members of arrays $P_3(i,j)$, $P_4(i,j)$ in the range (16;50) we have:

18,19,23,28,30,33,38,41,43,47,48- from $P_3(i,j)$;

21,27,28,35,40,42,49 - from $P_4(i,j)$

Indexes $P$ of members of the sequence $S_2(P) = 6P + 7$ that do not appear in these arrays are:

16,17,20,22,24,25,26,29,31,32,34,36,37,39,44,45,46,50

And Primes-in-S2 are:

103,109,127,139,151,157,163,181,193,199,211,223,229,241,271,277,283,307

We have found all primes in the range (100;300). So we have shown that there is no any mystery in the order of appearance of prime numbers and this order is fully determined by the properties of arrays (18)-(21).

## *3. C++ program based on matrix sieve algorithm*

In programming code were used following notations:

pr1= $P_{\min}$ ; pr2= $P_{\max}$;

R1[q], R2[r], S1[q], S2[r]-additional arrays corresponding to the range of P (pr1;pr2).

q- index of the arrays R1[q], S1[q].

r – index of the arrays R2[r], S2[r].

i2= $imax$; j1= $jmin$; j2= $jmax$;

P1=$P_1(i,j)$; P2=$P_2(i,j)$; P3=$P_3(i,j)$; P4=$P_4(i,j)$;

Expressions (18)-(21) for programming code can be rewritten as:

P1[i;j]=5+5*(i-1) + (7+ 6*(i-1))*(j-1) (18a)

P2[i;j]=5+7*(i-1) + (5+6*(i-1))*(j-1) (19a)

P3[i;j]=3+5*(i-1) + (5+6*(i-1))*(j-1) (20a)

P4[i;j]=7+7*(i-1) + (7+6*(i-1))*(j-1) (21a)

Using above equations (18a)-(21a) with area of change of i and j determined as before (22)-(26), the C++ program was developed to find prime numbers in the range ($N_1$;$N_2$) (Appendix 1).Program was successfully tested on the ordinary notebook up to N= 2 000 000 000 000 000 000, for this value run time equals 50 sec.

## 4. *Primality criteria*

Taking into account above stated considerations following criteria can be formulated:

Natural number $N$=6$P$+5; $P$=0, 1, 2, 3,… is a prime if and only if there is no integer solution for equation

$$P = 6xy - x + y - 1;\ x\geq1;\ y\geq1$$

Natural number $N$=6$P$+7; $P$=0, 1, 2, 3,… is a prime if and only if no one of two equations

$$P = 6xy - x - y - 1;\ x \geq 1; y \geq x$$

$$P = 6xy + x + y - 1;\ x \geq 1; y \geq x$$

has integer solution.

## 5. *Conclusion*

Leonhard Euler commented :"Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and I have reason to believe that it is a mystery into which the mind will never penetrate" [1].

We have different opinion: there is no any mystery at all and the distribution of prime numbers is fully determined by the properties of arrays (18)-(21) and can be summarized as:

Natural numbers that do not appear in arrays

$$6ij - i + j - 1 = \begin{pmatrix} 5 & 12 & 19 & 26 & 33 & 40 & \\ 0 & 23 & 36 & 49 & 62 & 75 & \dots \\ 0 & 0 & 53 & 72 & 91 & 110 & \dots \\ 0 & 0 & 0 & 95 & 120 & 145 & \dots \\ 0 & 0 & 0 & 0 & 149 & 180 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & 215 & \dots \end{pmatrix};\ i = 1,2,3\dots;\ j \geq i$$

$$6ij + i - j - 1 = \begin{pmatrix} 0 & 10 & 15 & 20 & 25 & 30 & \dots \\ 0 & 0 & 34 & 45 & 56 & 67 & \dots \\ 0 & 0 & 0 & 70 & 87 & 104 & \dots \\ 0 & 0 & 0 & 0 & 118 & 141 & \dots \\ 0 & 0 & 0 & 0 & 0 & 178 & \dots \\ . & . & . & . & . & 0 & \dots \end{pmatrix};\ i = 1,2,3,\dots;\ j \geq i + 1$$

are indexes $P$ of all primes in the sequence $S_1(P) = 6P + 5;\ P = 0,1,2,3,\dots$

Natural numbers that do not appear in arrays

$$6ij - i - j - 1 = \begin{pmatrix} 3 & 8 & 13 & 18 & 23 & 28 & \dots \\ 0 & 19 & 30 & 41 & 52 & 63 & \dots \\ 0 & 0 & 47 & 64 & 81 & 98 & \dots \\ 0 & 0 & 0 & 87 & 110 & 133 & \dots \\ 0 & 0 & 0 & 0 & 139 & 168 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & 203 & \dots \end{pmatrix} \;;\; i = 1,2; 3, \dots;\; j \geq i$$

$$6ij + i + j - 1 = \begin{pmatrix} 7 & 14 & 21 & 28 & 35 & 42 & \dots \\ 0 & 27 & 40 & 53 & 66 & 79 & \dots \\ 0 & 0 & 59 & 78 & 97 & 116 & \dots \\ 0 & 0 & 0 & 103 & 128 & 153 & \dots \\ 0 & 0 & 0 & 0 & 159 & 190 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & 227 & \dots \end{pmatrix} \;;\; i = 1,2,3, \dots;\; j \geq i$$

are indexes $P$ of all primes in the sequence $S_2(P) = 6P + 7;\ P = 0,1,2,3,\dots$

## *References*


1. Havil, J. Gamma: Exploring Euler's Constant. Princeton, NJ: Princeton University Press, 2003.

2. Maya Kaczorowski. The Evolution of Algorithms to find Prime Numbers.http://msurj.mcgill.ca/vol4/iss1/Kaczorowski2009.pdf.



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## Appendix 1

```
#include <cstdlib>

#include <iostream>

#include <math.h>

#include <ctime>
```

```
using namespace std;
 main(  )
{
                              /*  3S MATRIX  SIEVE*/
                             /*FINDING PRIMES IN THE RANGE (N1;N2)*/
                            /*Authors: B.Sklyar, D.Sklyar, I.Sklyar*/

                     /* N1>17;    N2<2 000 000 000 000 000 000; N2-N1<30000*/

                    unsigned long long  int N1=19527221980515000; unsigned long long  int N2 =19527221980515200;
                    unsigned long long  int pr1=floor(N1/6); unsigned long long  int pr2=ceil( N2/6);
              int r= 5000; int R2[r]; int rm=pr2-pr1;unsigned long long int S2[r];int r3;int r4;
              int q=5000; int R1[q] ; int qm=rm; unsigned long long int S1[q] ; int q2; int q1;
              for (q=1;q<qm;q++)
              R1[q] =1;
              for (r=1;r<rm;r++)
              R2[r] =1;
    unsigned long longint i; unsigned long long int j;
    unsigned long long int P1;unsigned long long int P2;
unsigned long long int P3;unsigned long long int P4;
unsigned long long int i2= sqrt( pr2/6)+2;
   long long int j1; long long int j2;
 unsigned long long int B;unsigned long long int K;
for ( i=1;i<i2;i++)
{ j2=(pr2+i+1)/( 6*i+1)+1;j1=(pr1+i+1)/( 6*i+1);
B=5+5*( i-1);  K=7+6*( i-1);
if ( i>j1) j1=i;
  for(j=j1; j<j2;j++)
```

```
{ P1=B+K*( j-1);
if(( P1>pr1)&&( P1<pr2))
 {  q1=P1-pr1; R1[ q1] =0;
   } }
    j2=(pr2-i+1)/( 6*i-1)+1;j1=(pr1-i+1)/( 6*i-1);
    if (j1<1) j1=1;
    B=5+7*( i-1);  K=5+6*( i-1);
     if ( i>j1-1) j1=i+1;
     for(j=j1; j<j2;j++)
{  P2=B+K*( j-1);

if(( P2>pr1)&&( P2<pr2))
    {      q2=P2-pr1; R1[ q2] =0;
  } }
         j2=(pr2+i+1)/( 6*i-1)+1;j1=(pr1+i+1)/( 6*i-1);
        B=3+5*( i-1);  K=5+6*( i-1);
    if ( i>j1) j1=i;
        for(j=j1; j<j2;j++)
{    P3=B+K*( j-1);
    if(( P3>pr1)&&( P3<pr2))
    {     r3=P3-pr1; R2[ r3]=0;
  }  }
       j2=(pr2-i+1)/( 6*i+1)+1;j1=(pr1-i+1)/( 6*i+1);
       B=7+7*(i-1);  K=7+6*(i-1);
     if ( i>j1)j1=i;
for(j=j1; j<j2;j++)
{ P4=B+K*( j-1);
   if(( P4>pr1)&&( P4<pr2))
   { r4=P4-pr1; R2[ r4] =0;
    } } }
```

```
    cout<<"i2="<<i2<<"    ;pr2="<<pr2<<"    \n";
    cout<<"P=pr1+q; pr1="<<pr1<<"   \n";
     for ( q=1;q<qm;q++) { S1[q] =R1[q]*((pr1+q)*6+5);  if (S1[q]%5==0) continue;
cout<<"q="<<q<<"; Prime in S1[P]=6*P+5="<<S1[ q]<<"  \n";}
cout<<"P=pr1+r; pr1="<<pr1<<"   \n";
     for ( r=1;r<rm;r++) { S2[r] =R2[r]*((pr1+r)*6+7);if (S2[r]%5==0) continue;
    cout<<"r="<<r<<"; Prime in S2[P]=6*P+7="<<S2[ r]<<"  \n";}
     cout<<clock();
  system("PAUSE");
 return EXIT_SUCCESS;

}
```