Attempt # 1

Since

$$|x\pm\rangle = \frac{1}{\sqrt{2}}|z+\rangle \pm \frac{1}{\sqrt{2}}|z-\rangle$$

$\therefore$

$$|x+\rangle = \frac{1}{\sqrt{2}}|z+\rangle + \frac{1}{\sqrt{2}}|z-\rangle$$

and

$$|x-\rangle = \frac{1}{\sqrt{2}}|z+\rangle - \frac{1}{\sqrt{2}}|z-\rangle$$

$$\hat{S}_x|x+\rangle = \frac{1}{\sqrt{2}}\left[|z+\rangle + |z-\rangle\right]$$

$$\hat{S}_x|x-\rangle = \frac{1}{\sqrt{2}}\left[|z+\rangle - |z-\rangle\right]$$

In matrix form, we can represent $\hat{S}_x$ as

$$\hat{S}_x = \frac{1}{\sqrt{2}}\begin{pmatrix} |z+\rangle + |z-\rangle & 0 \\ 0 & |z+\rangle - |z-\rangle \end{pmatrix}$$

$$\hat{S}_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & |z+\rangle + |z-\rangle \\ |z+\rangle + |z-\rangle & 0 \end{pmatrix}$$

$$\hat{S}_z = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & i|z+\rangle - i|z-\rangle \\ i|z+\rangle + i|z-\rangle & 0 \end{pmatrix}$$