

a)

$$\begin{aligned}
U^\dagger \begin{pmatrix} \xi_k & -\Delta \\ -\Delta^* & -\xi_k \end{pmatrix} U &= \begin{pmatrix} u_k^* & -v_k^* \\ v_k & u_k \end{pmatrix} \begin{pmatrix} \xi_k & -\Delta \\ -\Delta^* & -\xi_k \end{pmatrix} \begin{pmatrix} u_k & v_k^* \\ -v_k & u_k^* \end{pmatrix} \\
&= \begin{pmatrix} \xi_k(|u_k|^2 - |v_k|^2) + \Delta u_k^* v_k + \Delta^* v_k^* u_k & \Delta^* (v_k^*)^2 - \Delta (u_k^*)^2 + 2\xi_k u_k^* v_k^* \\ \Delta v_k^2 - \Delta^* u_k^2 + 2\xi_k v_k u_k & \xi_k(|v_k|^2 - |u_k|^2) - (\Delta u_k^* v_k + \Delta^* v_k^* u_k) \end{pmatrix} = \begin{pmatrix} E_+ & 0 \\ 0 & E_- \end{pmatrix}
\end{aligned}$$