

|  | Matrices                                    |   | Operators  |   |
|--|---|---|--|---|
|  | Real  | Complex                                       | Finite space<br>(All linear operators are bounded)   | Infinite Dimensions<br>(Not possible to use Matrices)   |
| Symmetric                                    | $A = A^T$<br>(is Normal)                    | $B = B^T$<br>BUT B might not be Normal        | $\equiv$ Self Adjoint  | $\langle Au v\rangle = \langle u Av\rangle$<br>Unbounded  |
| Anti-symmetric / skew symmetric (All Normal) | $A = -A^T$                                  |   |  |   |
| Self Adjoint                                 | $\equiv$ Symmetric                          | $\equiv$ Hermitian                            | $\langle Au v\rangle = \langle u Av\rangle$ (cf: Adjoint)<br>$A = A^\dagger$ OR $A = A^*$<br>(Real symmetric matrices represent Self-Adjoint operators)                    | (For the course, assume bounded in Hilbert space?)<br><br>Does that mean we stay in finite Hilbert space? |
| Hermitian (All Normal)                       | N/A (Symmetric)                             | Is Symmetric AND complex, AND $A = A^\dagger$ | self Adjoint AND satisfying boundary conditions  |   |
| Anti-Hermitian (All Normal)                  |   | $A = -A^\dagger$                              |  |   |
| Adjoint                                      | N/A   | $A^\dagger = (A^T)^*$                         | $\langle u \mathcal{L}u\rangle = \langle \overline{\mathcal{L}u} u\rangle$<br>$\langle f g\rangle = \langle g f\rangle^*$ OR $\langle f Tg\rangle = \langle T^*f g\rangle$ | (For the course, assume bounded in Hilbert space?)  |
| Unitary (All Normal)                         | $U^{-1} = U^T$<br>(Real U's are orthogonal) | $U^{-1} = U^\dagger$                          |  |   |
| Conjugate                                    | N/A   | $A^* = a_{ij}^*$                              | Hermitian Conjugate $\equiv$ Self Adjoint  |   |
| Conjugate Transpose                          | $A^T$                                       | $A^\dagger = (A^T)^*$                         |  |   |

Key: conjugate(A) =  $A^*$  ; transpose(A) =  $A^T$  ; conjugate transpose(A) =  $A^\dagger$

- Matrix is normal if  $[B, B^*] = 0$ , ie  $B, B^*$  commute