TT Ball Trajectory

$dia := 0.041$		Diameter of a ping pong ball in meters
$r := \frac{dia}{2}$	$r = 0.0205$	Radius of a ping pong ball in meters
$A := \pi \cdot r^2$	$A = 0.00132$	Area of cross section of a ping pong ball in m^2.
$mass := 0.0027$		Mass of a ping pong ball in kg
$I := \frac{2 \cdot mass \cdot r^2}{3}$	$I = 7.5645 \times 10^{-7}$	Kg*m^2
$\rho := 1.2$		Density of dry air in kg/m^3 at sea level
$C_d := 0.5$		Ideal drag coefficient for a sphere. Unitless
$C_m := 0.29$		Magnus coefficient
$\eta := 1.78 \cdot 10^{-5}$		Viscosity of air in $\frac{kg}{m \cdot s}$
$b := 6 \cdot \pi \cdot \eta \cdot r$	$b = 6.878203 \times 10^{-6}$	Viscous friction coefficient.

Magnus combined constant

$M := 4 \cdot \pi \cdot C_m \cdot r^3 \cdot \rho$	$M = 0.000038$	From a document found on line
$M := \frac{4}{3} \pi r^3 \cdot \rho$	$M = 0.000038$	From Wikipedia. The two versions are almost the same.
$v_0 = 54$		Initial velocity in m/s
$\theta = 10\,deg$	$\theta = 0.174533$	Angle relative to horizontal
$\frac{\omega_0}{2 \cdot \pi} = 75$	$\omega_0 \cdot r = 9.660397$	Initial spin in radians per second and tangential velocity

$$Q := \begin{pmatrix} x \leftarrow -1.5 \\ x' \leftarrow v_0 \cdot \cos(\theta) \\ y \leftarrow 0.05 \\ y' \leftarrow v_0 \cdot \sin(\theta) \\ \omega \leftarrow \omega_0 \end{pmatrix}$$

Initial horizontal and vertical positions and velocities and spin

$$D(t, Q) := \left| \begin{array}{l} \begin{pmatrix} x \\ x' \\ y \\ y' \\ \omega \end{pmatrix} \leftarrow Q \\ x'' \leftarrow -\dfrac{b \cdot x'}{mass} - \operatorname{sign}(x') \cdot \dfrac{\rho \cdot C_d \cdot A \cdot \dfrac{x'^2}{2}}{mass} + \dfrac{M \cdot y' \cdot \omega}{mass} \\ y'' \leftarrow -9.8 - \dfrac{b \cdot y'}{mass} - \operatorname{sign}(y') \cdot \dfrac{\rho \cdot C_d \cdot A \cdot \dfrac{y'^2}{2}}{mass} - \dfrac{M \cdot x' \cdot \omega}{mass} \\ \omega' \leftarrow -0.03 \cdot \omega \\ \begin{pmatrix} x' \\ x'' \\ y' \\ y'' \\ \omega' \end{pmatrix} \end{array} \right.$$

Break out the state variables into horizontal position, horizontal velocity, vertical position, vertical velocity and radians per second

Compute horizontal deceleration

Computes vertical acceleration/deceleration

Slow down the spin of the ball exponentially. Slow down 3% per second. This seems slow but I have seen ball spin for many seconds.

Return rate of change in the state

TT Ball Trajectory

$SimTime := 0.2$ — Simulation time

$\Delta t := 0.00025$ — Time increment

$N := \frac{SimTime}{\Delta t}$ $\quad N = 800$ — Number of time increments

$Z := rkfixed(Q, 0, SimTime, N, D)$ — Integrate using Runge-Kutta

$Z =$

	Time 0	x 1	x' 2	y 3	y' 4	ω 5
0	0	-1.5	53.179619	0.05	9.377002	471.238898
1	0.00025	-1.486716	53.091379	0.052333	9.284005	471.235364
2	0.0005	-1.473454	53.00333	0.054642	9.191218	471.23183
3	0.00075	-1.460214	52.91547	0.056928	9.098638	471.228295
4	0.001	-1.446997	52.827799	0.059191	9.006265	471.224761
5	0.00125	-1.433801	52.740316	0.061431	8.914098	471.221227
6	0.0015	-1.420626	52.653019	0.063648	8.822135	471.217693
7	0.00175	-1.407474	52.565908	0.065842	8.730376	471.214159
8	0.002	-1.394343	52.478981	0.068014	8.638819	471.210625
9	0.00225	-1.381235	52.392239	0.070162	8.547463	471.20709
10	0.0025	-1.368147	52.30568	0.072287	8.456308	471.203556
11	0.00275	-1.355082	52.219302	0.07439	8.365352	471.200022
12	0.003	-1.342038	52.133106	0.07647	8.274595	471.196488
13	0.00325	-1.329015	52.047089	0.078527	8.184035	471.192954
14	0.0035	-1.316014	51.961253	0.080562	8.093671	471.189421
15	0.00375	-1.303034	51.875594	0.082574	8.003502	471.185887

$t := Z^{\langle 0\rangle}$ $\quad x := Z^{\langle 1\rangle}$ $\quad x' := Z^{\langle 2\rangle}$ $\quad y := Z^{\langle 3\rangle}$ $\quad y' := Z^{\langle 4\rangle}$ $\quad \omega := Z^{\langle 5\rangle}$

$$i := \left| \begin{array}{l} i \leftarrow 0 \\ \text{while } Z_{i,3} > 0 \\ \quad i \leftarrow i + 1 \end{array} \right.$$

$i = 256$

$x_i = 1.280147$ $\quad y_i = -0.00034$ $\quad \omega_i = 470.334987$

$x'_i = 34.867845$ $\quad y'_i = -9.460708$

$n := 0..i$

$x := submatrix(x, 0, i, 0, 0)$ — extract the horizontal positions from Z

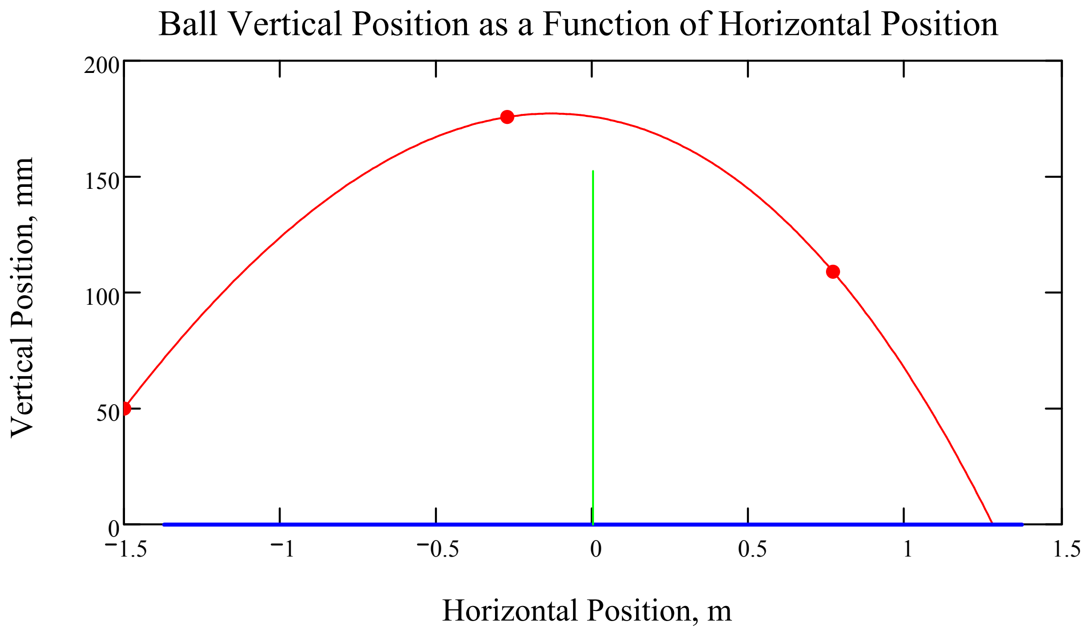
$y := submatrix(y, 0, i, 0, 0)$ — extract the vertical positions from Z

$table_n := 0$ — table height is 0

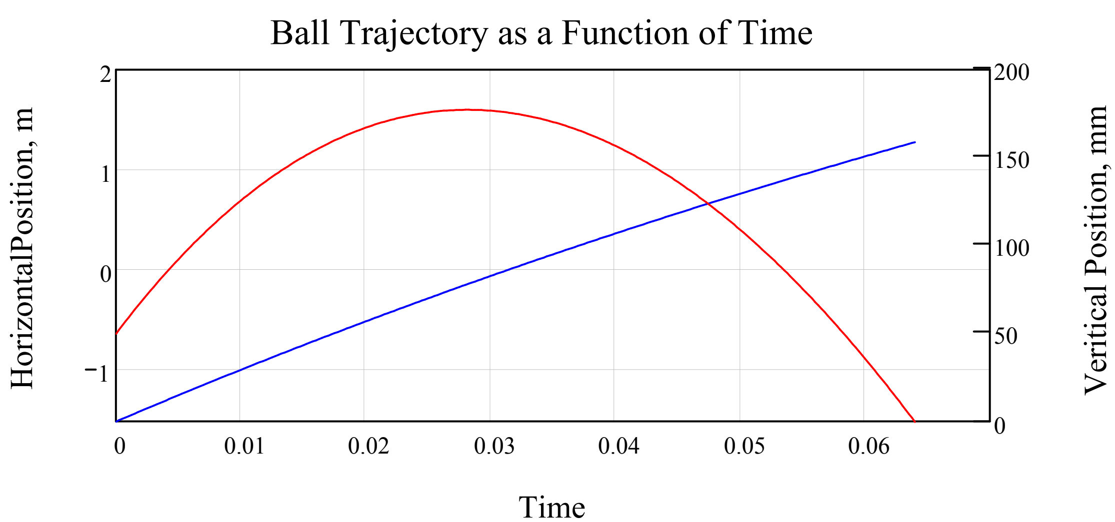
$net_n := 0.1525$ — Net height in meters

$tbl\_len := 9 \cdot ft$ $\quad tbl\_len = 2.7432\,m$ — Table length

$tbl\_end := \frac{tbl\_len}{2}$ $\quad tbl\_end = 1.3716\,m$ — Distance from net. + for the far end. - for the near end



●●● Ball Position, dots every 25 msec
Table Surface Height
Net Height



Horizontal Position
Vertical Position

$v_0 \equiv 54$ $\theta \equiv 10 \cdot deg$ $\omega_0 \equiv 2 \cdot \pi \cdot 75$ $i \cdot \Delta t = 0.064$ seconds

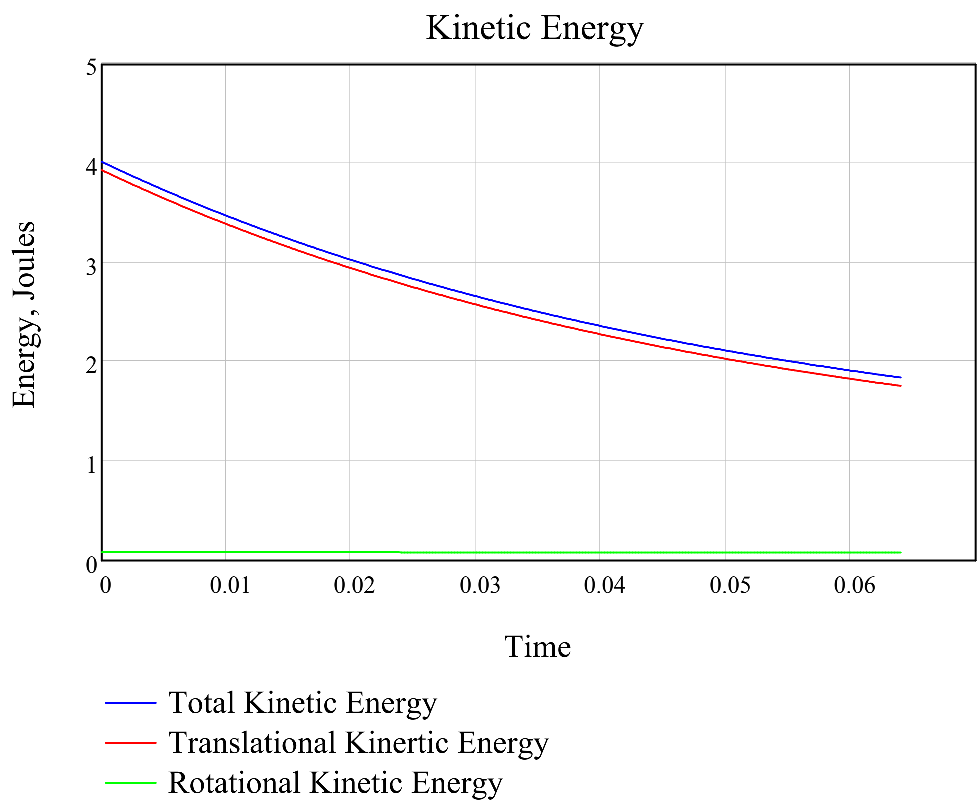
Kinetic Energy

$$Er_n := \frac{1}{2} \cdot I \cdot \left(\omega_n\right)^2$$

Rotational Kinetic Energy
I is the rotational inertia and
ω_n angular velocity in rad/s

$$Et_n := \frac{1}{2} \cdot mass \cdot \left[\left(x'_n\right)^2 + \left(y'_n\right)^2\right]$$

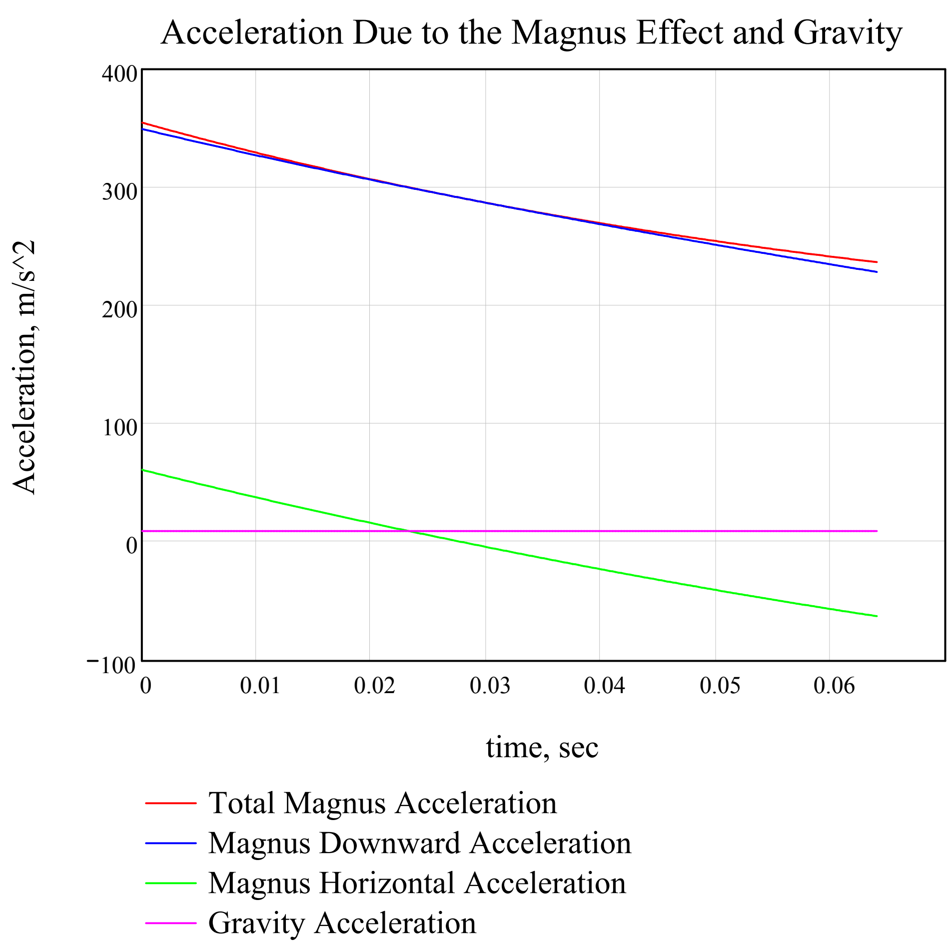
Translational Kinetic Energy
x` is the horizontal velocity
y` is the vertical velocity



Accelerations Due to the Magnus Effect

$$Mx_n := \frac{M \cdot y'_n \cdot \omega_n}{mass}$$

$$My_n := \frac{M \cdot x'_n \cdot \omega_n}{mass}$$



TT Ball Trajectory

Verify Units

$dia := 0.041 \cdot m$		TT ball diameter
$r := \frac{dia}{2}$	$r = 0.0205\,m$	TT ball radius
$A := \pi \cdot r^2$	$A = 0.00132\,m^2$	TT ball cross sectional area
$mass := 0.0027 \cdot kg$		TT ball mass
$I := \frac{2 \cdot mass \cdot r^2}{3}$	$I = 7.5645 \times 10^{-7}\,m^2 \cdot kg$	TT ball inertia
$\rho := 1.225 \cdot \frac{kg}{m^3}$		Density of air
$C_d := 0.5$		Coefficient of drag
$C_m := 0.29$		Magnus effect coefficient. Wikipedia says 0.33333
$\eta := 1.78 \cdot 10^{-5} \cdot \frac{kg}{m \cdot s}$		Viscous friction coefficient
$b := 6 \cdot \pi \cdot \eta \cdot r$		Combine viscous friction coefficient
$M := 4 \cdot \pi \cdot C_m \cdot r^3 \cdot \rho$	$M = 0.000038\,kg$	Combined Magus coefficient
$v_0 := 50 \cdot \frac{m}{s}$		Initial velocity, combined x and y
$\theta := 12 \cdot deg$	$\theta = 0.20944$	Trajectory angle about horizontal
$\omega_0 := 2 \cdot \pi \cdot 75 \cdot \frac{rad}{s}$		Initial spin
$\omega := \omega_0$		
$x' := v_0 \cdot \cos(\theta)$	$x' = 48.90738\,\frac{m}{s}$	Horizontal velocity
$y' := v_0 \cdot \sin(\theta)$	$y' = 10.395585\,\frac{m}{s}$	Vertical velocity

Acceleration and Units

All the terms result in an acceleration in meters per second squared to the units are consistent.

Horizontal acceleration. In this case it is negative or decelerating.

$$x'' := -\frac{b \cdot x'}{mass} - \text{sign}(x') \cdot \frac{\rho \cdot C_d \cdot A \cdot \frac{x'^2}{2}}{mass} + \frac{M \cdot y' \cdot \omega}{mass} \qquad x'' = -288.538799 \, \frac{m}{s^2}$$

Vertical acceleration. Negative values mean downwards

$$y'' := -g - \frac{b \cdot y'}{mass} - \text{sign}(y') \cdot \frac{\rho \cdot C_d \cdot A \cdot \frac{y'^2}{2}}{mass} - \frac{M \cdot x' \cdot \omega}{mass} \qquad y'' = -354.30619 \, \frac{m}{s^2}$$

All the individual terms have consistent units of acceleration.

$$\frac{b \cdot x'}{mass} = 0.124591 \, \frac{m}{s^2}$$

Viscous damping

$$\text{sign}(x') \cdot \frac{\rho \cdot C_d \cdot A \cdot \frac{x'^2}{2}}{mass} = 358.194345 \, \frac{m}{s^2}$$

Drag

$$\frac{4 \cdot \pi \cdot C_m \cdot r^3 \cdot \rho \cdot x' \cdot \omega}{mass} = 328.289732 \, \frac{m}{s^2}$$

Magus effect