

Name Robb Connell Section: a.m. p.m.

Elementary Linear Algebra (MATH 2300)
Quiz #6

Instructions. Show or explain all of your work.

1. Let U be the space of polynomials with basis $E = [1, t, t^2]$, let $V \cong \mathbb{R}^2$ have the basis $F = [(1, 0), (1, 1)]$, and define a linear map $U \xrightarrow{T} V$ by $T(f) = (f(3), f'(3))$.

(a) [4] Determine the matrix representing T relative to the bases E and F . What is the rank of T ?

$$\left[c_F \left(\frac{d}{dt}(1) \right) \quad c_F \left(\frac{d}{dt}(t) \right) \quad c_F \left(\frac{d}{dt}(t^2) \right) \right]$$
$$= \left[c_F(0) \quad c_F(1) \quad c_F(2t) \right]$$

$$T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{rank}(T) = 2$$

(b) [2] Exhibit a basis for the kernel of T .

2. [4] Suppose a subspace $W \subset \mathbb{R}^4$ is spanned by $(1, 1, 3, 4)$ and $(1, 2, 1, 2)$. Exhibit a basis for the subspace $W^\perp \subset \mathbb{R}^4$ orthogonal to W .

$$w_1 = (1, 1, 3, 4)$$

$$w_2 = (1, 2, 1, 2) - \left[\langle (1, 1, 3, 4), (1, 2, 1, 2) \rangle \langle (1, 1, 3, 4), (1, 1, 3, 4) \rangle \right] (1, 1, 3, 4)$$
$$= (1, 2, 1, 2) - \left(\frac{14}{27}, \frac{14}{27}, \frac{42}{27}, \frac{56}{27} \right)$$
$$= \left(\frac{13}{27}, \frac{40}{27}, -\frac{15}{27}, -\frac{2}{27} \right) = (13, 40, -15, -2)$$

$$\text{basis: } \left\{ (1, 1, 3, 4), (13, 40, -15, -2) \right\}$$