MAT1163D: LINEAR ALGEBRA
ASSIGNMENT 1
DUE DATE: 28th March 2013, at 13:30

The purpose of this assignment is to enable students to develop and demonstrate their knowledge, understanding and skills by solving problems from the unit content.

Submission Guidelines:

1. Full marks will be gained by complete solutions to all questions. Total marks for each question are given. Where a question consists of more than one part the parts need not carry equal marks.

2. Answer the questions in the order given.

3. Be clear and concise. Show all of your working. All handwriting must be legible. If using a word processor, ensure correct mathematical symbols and layout. Answers to the assignment problems need to be written up in grammatically correct English. Failure to do so will result in deduction of marks.

4. The assignment solutions must be your own work. Each submission from your assignment must have a completed coversheet attached and be submitted on or before the due date. You are reminded of the declaration:

   "I certify that the attached assignment is my own work and that any material drawn from other sources has been acknowledged".

   that you sign when you complete the coversheet.

5. In the interests of fairness, extension of time for submission of the assignment questions will be given only in exceptional circumstances, and then only in accordance with PIBT rules.

6. You may hand in your completed assignment by giving it to the lecturer.

7. This assignment contributes 15% to the total mark for this unit.

Question 1 [7 marks]

The reduced row echelon form of $A = \begin{bmatrix} 1 & 2 & 5 & b \\ 4 & a & 17 & -22 \\ & row3 & & \end{bmatrix}$ is equal to $R = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) What can you say about row 3 of A? Give an example of a possible third row for A.
(b) Determine the values of a and b.
(c) Determine the solution of the homogeneous system of equations $R\mathbf{x} = \mathbf{0}$ in parametric vector form.
(d) What is the dimension of the column space of A? Do the columns of A span $\mathbf{R}^3$?

Question 2 [6 marks]

Suppose an economy has four sectors, Agriculture (A), Energy (E), Manufacturing (M), and Transportation (T). Sector A sells 10% of its output to E and 25% to M and retains the rest. Sector E sells 30% of its output to A, 35% to M, and 25% to T and retains the rest. Sector M sells 30% of its output to A, 15% to E, and 40% to T and retains the rest. Sector T sells 20% of its output to A, 10% to E, and 30% to M and retains the rest.

(a) Construct a diagram that shows the exchange between the three sectors.
(b) Determine the exchange table for this economy, where the columns describe how the output of each sector is exchanged among the four sectors.
(c) Denote the prices of the total annual outputs of the sectors by p_A, p_E, p_M and p_T respectively. Determine the equations that need to hold for the equilibrium prices for the four sectors.
(d) Find the equilibrium prices, if they exist.

Question 3 [6 marks]

Balance the following chemical equation using the vector equation approach.

$PbN_6 + CrMn_2O_8 \rightarrow Pb_3O_4 + Cr_2O_3 + MnO_2 + NO$

Question 4 [6 marks]

Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -5 \\ -3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -2 \\ 10 \\ 6 \end{bmatrix}$ and $\mathbf{v}_3 = \begin{bmatrix} 2 \\ -9 \\ h \end{bmatrix}$.

(a) For what values of h is $\mathbf{v}_3$ in Span $\{\mathbf{v}_1, \mathbf{v}_2\}$? Justify your answer.
(b) For what values of h is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly dependent? Justify your answer.

Question 5 [10 marks]

Consider the linear transformation $T(x_1, x_2, x_3) = (2x_1 - 2x_2 - 4x_3, x_1 + 2x_2 + x_3)$

(a) Find the image of $(3, -2, 2)$ under T.

(b) Does the vector (5, 3) belong to the range of T?

(c) Determine the matrix of the transformation.

(d) Is the transformation T onto? Justify your answer

(e) Is the transformation one-to one? Justify your answer

Question 6 [5 marks]

a) For each of the following matrices explain why the matrix is not invertible.

i) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix}$ ii) $\begin{bmatrix} 3 & 4 & -6 \\ 7 & 2 & 1 \\ -6 & -8 & 12 \end{bmatrix}$ iii) $\begin{bmatrix} 5 & -2 & 15 \\ 1 & -4 & 3 \\ 2 & 1 & 6 \end{bmatrix}$

b) Suppose A is an $n \times n$ matrix with the property that the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution. Without using the Invertible Matrix Theorem, explain directly why the equation $A\mathbf{x} = \mathbf{b}$ must have a solution for each $\mathbf{b}$ in $\mathbf{R}^n$