where we collected the linear and nonlinear (in the a_μ fields) terms separately leaving only terms corresponding to the expansion in the Higgs mode $n \cdot A$, as is taken in Eq.(8), and also retained the former notation for fermion ψ. We take the Greek letters for the Lorentz indices ($\mu, \nu, \rho, \sigma = 0, 1, 2, 3$) and the metric is $g_{\mu\nu} = (1, -1, -1, -1)$, while everywhere when appears $(n^2)^2$ (and higher powers of n^2) we replace it by 1. For the photon-electron and photon-photon interactions it follows then in the lowest approximation

$$\mathcal{L}_{NL}^{int} = -ea_\mu \overline{\psi}\gamma^\mu\psi + e\frac{n^2 a_\rho^2}{2M}\bar{\psi}(\gamma \cdot n)\psi - \frac{n^2}{M}(\partial_\mu a_\nu n^\mu a_\rho \partial^\nu a^\rho) - \frac{1}{16M^2}\left[(n^\mu\partial^\nu - n^\nu\partial^\mu)\, a_\rho^2\right]^2 \tag{11}$$

The Lagrangian (10) together with the gauge fixing condition (4) completes the nonlinear σ model type construction for quantum electrodynamics. We call this the nonlinear QED. The model contains the massless vector Goldstone boson modes and keeps the massive Higgs mode frozen, and in the limit $M \to \infty$ the model (given just by the first line in the Lagrangian $\mathcal{L}_{NL}$ (10)) is indistinguishable from conventional QED taken in the temporal or axial gauge (4). So, for this part of the Lagrangian $\mathcal{L}_{NL}$ the spontaneous LIV only means the noncovariant gauge choice (4) in otherwise the gauge invariant (and Lorentz invariant) theory. However, we will show in the next section that also all other terms in the $\mathcal{L}_{NL}$ (10), though being by themselves the Lorentz and $C(CPT)$ violating ones, cause no the physical LIV effects at least in the one-loop approximation.

2.2 The Feynman rules

They for the interaction Lagrangian $\mathcal{L}_{NL}^{int}$ (11) include:

i/ An ordinary QED photon-electron vertex is

$$-ie\gamma_\mu \tag{12}$$

ii/ The contact 2-photon-electron vertex is given by

$$i\frac{eg_{\mu\nu}n^2}{M}(\gamma \cdot n) \tag{13}$$

iii/ The 3-photon vertex (with photon 4-momenta k_1, k_2 and k_3) is appeared as

$$-\frac{in^2}{M}\left[(k_1 \cdot n)k_{1,\alpha}g_{\beta\gamma} + (k_2 \cdot n)k_{2,\beta}g_{\alpha\gamma} + (k_3 \cdot n)k_{3,\gamma}g_{\alpha\beta}\right] \tag{14}$$

where the second index in the each momentum k_1, k_2 and k_3 denotes its Lorentz component;

iv/ The 4-photon vertex (with photon 4-momenta k_1, k_2, k_3 and k_4) is

$$\begin{aligned}&\frac{-i}{M^2}\{[(n^2(k_1+k_2)^2 - (n(k_1+k_2))^2]g_{\alpha\beta}g_{\mu\nu} + [n^2(k_1+k_3)^2 - (n(k_1+k_3))^2]g_{\alpha\mu}g_{\beta\nu} + \\ &+[n^2(k_1+k_4)^2 - (n(k_1+k_4))^2]g_{\alpha\nu}g_{\beta\mu}\}\end{aligned} \tag{15}$$

v/ The photon propagator is in general (for $n^\mu a_\mu = 0$)

$$D_{\mu\nu}(k) = \frac{-i}{k^2 + i\epsilon}\left(g_{\mu\nu} - \frac{n_\mu k_\nu + k_\mu n_\nu}{n \cdot k} + \frac{n^2 k_\mu k_\nu}{(n \cdot k)^2}\right) \tag{16}$$