

# Calculation of the Optimum Installation Angle for Fixed Solar-Cell Panels Based on the Genetic Algorithm and the Simulated-Annealing Method

Yaow-Ming Chen, *Member, IEEE*, Chien-Hsing Lee, *Member, IEEE*, and Hsu-Chin Wu

**Abstract**—This paper presents the calculation of the optimum installation angle for the fixed solar-cell panels based on the genetic algorithm (GA) and the simulated-annealing (SA) method. The output power of the solar-cell panel is highly affected by the sunlight incident angle and its efficiency can be improved if the solar-cell panel is properly installed with the optimum angle. The relationship between the sunlight incident angle and the sunlight radiation intensity on the solar-cell panel surface is also presented in this paper. Both GA and SA with climatic data are utilized to calculate the optimum installation angle of the solar-cell panel for different locations in Taiwan. The best monthly and annual installation angles obtained by computer simulations are presented. Hardware experimental results indicate that the actual best monthly installation angles are very close to the computer simulation results.

**Index Terms**—Genetic algorithm (GA), photovoltaic power systems, simulated-annealing (SA) method, solar-cell panels.

## I. INTRODUCTION

THE applications of renewable energy power generation systems have been increased significantly over the past decade [1]–[3]. Among those renewable sources, solar energy has the advantages of no rotational parts, no atmospheric pollutions, and very low operation and maintenance costs [4]. Solar energy can be directly converted into electricity through different types of semiconductors with the photovoltaic (PV) effect. These semiconductors are called solar cells or PV cells. The applications of stand-alone or grid-connected PV power generation systems are very common in recent years.

The equivalent circuit of a solar cell, which consists of a current source in parallel with a resistor-diode network, is shown in Fig. 1.

The output current of the solar cell can be expressed as

$$I_o = I_{ph} - I_d \quad (1)$$

$$I_d = I_{sat} \left[ \exp \left( \frac{qV}{kT} \right) - 1 \right] \quad (2)$$

Manuscript received December 3, 2002; revised January 15, 2004. This work was supported in part by the National Science Council of Taiwan, R.O.C., under Project NSC 89-2218-E-194-031. Paper no. TEC-00266-2002.

Y.-M. Chen is with the Department of Electrical Engineering, National Chung Cheng University, Chiayi 621, Taiwan, R.O.C. (e-mail: ieeymc@ccu.edu.tw).

C.-H. Lee is with the Department of Electrical Engineering, National Yunlin University of Science and Technology, Yunlin 640, Taiwan, R.O.C. (e-mail: chlee@yuntech.edu.tw).

H.-C. Wu is with the Department of Electrical Engineering, I-Shou University, Kaohsiung 840, Taiwan, R.O.C.

Digital Object Identifier 10.1109/TEC.2004.832093

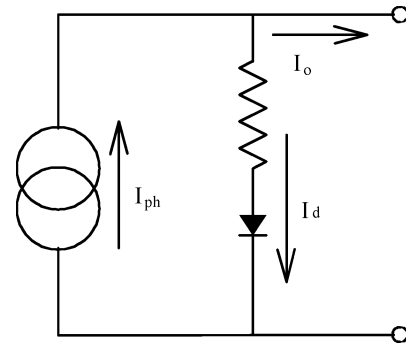


Fig. 1. Equivalent circuit of a solar cell.

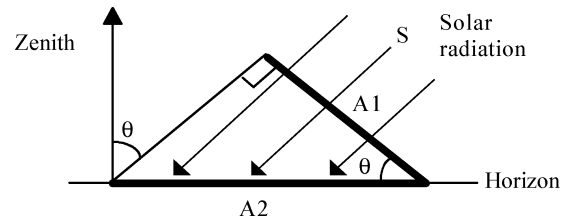


Fig. 2. Photon density of surface A1 and A2.

where

$I_{ph}$  photo-induced current;  
 $I_{sat}$  diode reverse saturation current;  
 $V$  voltage;  
 $q$  magnitude of the electron charge;  
 $k$  Boltzmann constant;  
 $T$  absolute temperature.

The photo-induced current, or called the generation current, is proportional to the number of photons that can be collected on the surface area of the solar cell. Since the objective of the solar-cell panel is to convert solar energy into electric energy, the intensity of the solar radiation on the solar-cell panel becomes a very important factor for the efficiency of the PV power generation system. Assuming the same solar radiation,  $S$ , and neglecting the reflection factor, the photon density of surface A1,  $D_{ph,A1}$ , and surface A2,  $D_{ph,A2}$ , as shown in Fig. 2 have the following relationships:

$$D_{ph,A1} = \frac{S}{A1} \quad (3)$$

$$D_{ph,A2} = \frac{S}{A2} \quad (4)$$

$$\frac{D_{ph,A2}}{D_{ph,A1}} = \cos \theta \leq 1 \quad (5)$$



Equations (7), (8), (11), and (13) show that the value of the angle between the incident sunlight and the normal line of the solar-cell panel in the 3-D space can be determined by the values of  $\theta_\beta$ ,  $\theta_\psi$ , and  $\theta_e$ . The complement to  $90^\circ$  of the view angle  $\theta_\beta$  is the solar-cell panel installation angle  $\theta$ , which is the user-controlled variable.

The azimuth  $\theta_\psi$  can be determined by the following equation [3]:

$$\theta_\psi = \cos^{-1} \left[ \frac{\sin \theta_e \cdot \sin \phi - \sin \delta}{\cos \theta_e \cdot \cos \phi} \right] \quad (14)$$

where  $\phi$  is the geographical latitude of the location and  $\delta$  is the solar declination at a particular time. The simple and commonly used formula for solar declination in degrees is given as [5]

$$\delta = \sin^{-1} \left\{ 0.4 \sin \left[ \frac{360}{365} (d_n - 82) \right] \right\} \quad (15)$$

where  $d_n$  is the number of days in a year, ranging from 1 on 1 January to 365 on December 31.

The incident angle  $X$  of the sunlight to the surface of the solar-cell panel at a different time of the day and different day of the year can be calculated by the series equations derived above. Since the photon density of the sunlight is directly related to the incident angle, there exists an optimal install angle for the solar-cell panel to collect a maximum number of photon on the surface of the solar-cell panel. On the other hand, the best installation angle for fixed solar-cell panels is highly related to the weather conditions of different locations; it is very difficult to obtain the mathematical model of this problem and use the gradient-descent algorithm to solve it. Hence, a new approach for determining the optimum installation angle of the solar-cell panel is needed.

### III. OVERVIEW OF THE GA AND THE SA METHOD

#### A. GA

The GA is a search mechanism based on the principle of nature selection and population genetics. The primary framework of the GA was first proposed in the book *Adaptation in Natural and Artificial Systems* by Dr. John Holland in 1975. There are many advantages that make the GA attractive [6]–[9]. It does not require the use of derivatives. Also, it has less computational burden than the gradient decent methodology and without the divergence problem. Instead of a point-by-point search, the GA offers a parallel search of the solution space rather than a single region. Hence, the GA can find a near optimal solution for a complex problem very quickly. This is good for an application that does not need a very accurate solution.

At the beginning of the GA, representations for possible solutions, which are often called chromosomes or individuals, must be developed. Different combinations of genes form different chromosomes. Each chromosome is a possible solution of the problem. The set of chromosomes is called the population of the generation. Chromosomes in a generation are forced to evolve toward better ones in the next generation by three basic GA operators, reproduction, crossover and mutation, and the problem-specified fitness function.

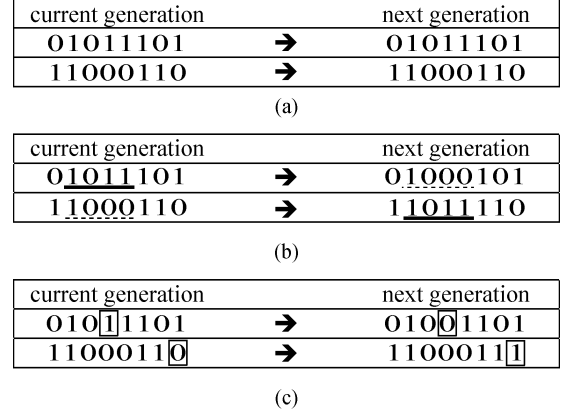


Fig. 4. Three basic GA operators. (a) Reproduction. (b) Crossover. (c) Mutation.

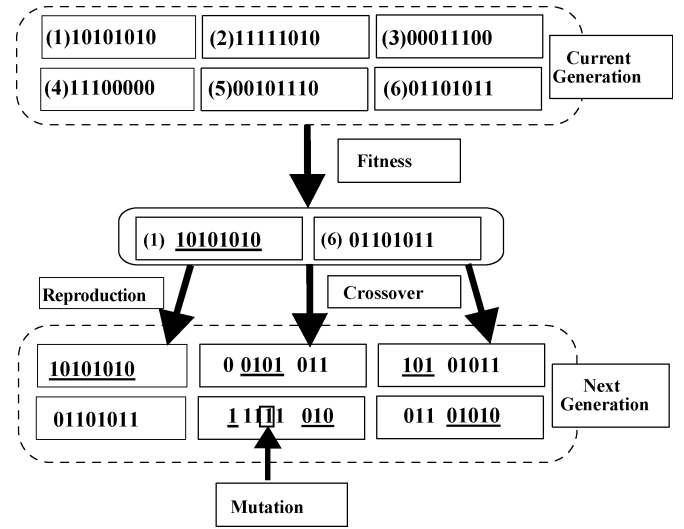


Fig. 5. Standard procedure of GA from one generation to the next generation.

In reproduction, a number of selected exact copies of chromosomes in the current population become a part of the offspring. In crossover, randomly selected subsections of two individual chromosomes are swapped to produce the offspring. In mutation, randomly selected genes in chromosomes are altered by a probability that is equal to the specified mutation rate. For a binary coding gene, it means that digit 1 becomes digit 0 and vice-versa. Fig. 4(a)–(c) are the schematic illustrations of the three basic GA operators for chromosomes with 8 bits of genes.

The standard procedure of GA operations from one generation to the next one is shown in Fig. 5. Each chromosome of the current population is evaluated by the fitness function and some good chromosomes are selected. If the stopping criterion is satisfied, then stop the program. Otherwise, a new population will be generated by using the three basic GA operators (i.e., reproduction, crossover, and mutation).

In this paper, the solar-cell panel installation angle is transferred into the binary gene code and the fitness function is defined as the sum of the sunlight intensity upon the solar-cell panel for a typical period of time. The local climatic condition, such as the sunshine duration, will affect the annual solar energy summation and will also affect the optimal installation angle

of the solar-cell panel too. Hence, five years of weather data provided by the Central Weather Bureau of Taiwan are adopted to obtain more accurate results. With the help of GA, the best monthly and annual installation angle for the maximum output power of the solar-cell panel in different locations can be obtained.

### B. SA Method

The SA method was first introduced by Metropolis *et al.* and then used to solve combinational optimization problems by Kirkpatrick *et al.* [10], [11]. Basically, annealing is the physical process of applying heat to a solid until it melts. The melted solid is then allowed to cool slowly in a controlled manner and the free energy of solid will have to be minimized. When the temperature is low enough, the solid will crystallize into a perfect lattice.

Although the SA method is essentially a random search method similar to the iterative improvement algorithm, it may accept an inferior solution based upon a probabilistic measurement. Therefore, it is possible to escape from being trapped within local minimum points and to seek after the global or near global in the overall solution space.

In this application, the temperature represents the installation angle of the solar-cell panel. In applying the SA method to solve the optimization problem, the cooling schedule includes the following three components [12], [13]:

- 1) initial temperature: the initial value of temperature must be large enough to find the optimum solution for a constrained problem;
- 2) temperature decreasing coefficient  $\alpha$ : the current temperature is lowered according to  $T_{k+1} = \alpha \times T_k$ . The coefficient  $\alpha$  is constant and is less than 1 (e.g.,  $\alpha$  is 0.8 to 0.99). When  $\alpha$  is equal to 0.99, it can find a closer optimal solution. On the other hand, when  $\alpha$  is equal to 0.8, it will speed up the convergence process;
- 3) final temperature: when the current temperature reaches the final temperature, the annealing process can be considered frozen and the optimal configuration has been achieved.

With the application of the SA technique, the procedure of computing the installation angle for the fixed solar-cell panel is described as follows:

- 1) start from an initial installation angle schedule;
- 2) calculate the sum of the incident sunlight intensity into the solar-cell panel for certain duration;
- 3) generate a new installation angle schedule by perturbing the current schedule while satisfying all of the constraints;
- 4) calculate the new sum of the incident sunlight intensity associated with the new schedule.
- 5) accept the new schedule only
  - a) if the new sum of the incident sunlight intensity is higher;
  - b) if the new sum of the incident sunlight intensity is lower, but the acceptance probability is higher than a randomly generated number between 0 and 1.
- 6) repeat steps 3)–5) for a specific number of trials;

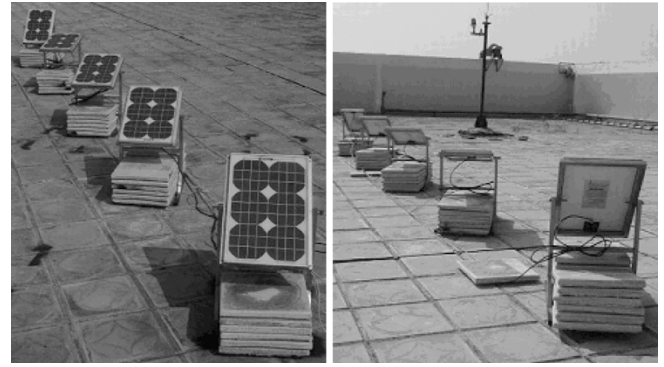


Fig. 6. Hardware configuration of the experienced solar-cell panels.

- 7) if the final temperature or the maximum number of iteration is reached, then stop the program. Otherwise, go back to step 3).

### IV. CONFIGURATION OF HARDWARE

A simple monitoring system is used in this experiment to illustrate the corresponding relationship between the actual installation angle of the fixed solar-cell panel and the simulated results. It consists of five sets of PV power systems and a voltage measurement circuit. Each PV power system includes a solar-cell panel and a resistor load of  $25\ \Omega$ . The generated solar power of the five tilted solar panels was recorded everyday. Then, the best installation angle of solar-cell panel can be observed each month.

The monocrystalline silicon solar-cell panel, SP10, manufactured by Siemens in Germany is used for a hardware experiment. It has a rated power of 10.5 W, an open voltage of 21.3 V, and a short current of 0.67 A. As shown in Fig. 6, five solar-cell panels with five different fixed installation angles were carefully arranged on the roof of the building to collect the solar energy.

Fig. 7 shows the block diagram of the voltage measurement circuit. Its specific function of each block is described as follows.

- 1) Control program: the control program of the data-acquisition system is implemented by using C language. Every 5 min, it will send out a series of sequential commands to measure and record data from the voltage measurement circuit. By monitoring the voltage value on the  $25\text{-}\Omega$  load continuously, the power produced by each solar-cell panel with a specific installation angle can be calculated.
- 2) 8255 input/output (I/O) interface: it can output the control signal and input any data.
- 3) TTL7445 decoder/driver: The binary digital control signals from the 8255 I/O interface are decoded in order to drive the selected relay to change the corresponding measurement point.
- 4) Signal switch circuit: It consists of five relays which are controlled by TTL7445. The selected conducted relay will enable the voltage measurement of a specific solar-cell panel.
- 5) Signal-ratio circuit: It is implemented by operational amplifiers with appropriate auxiliary circuit components. In

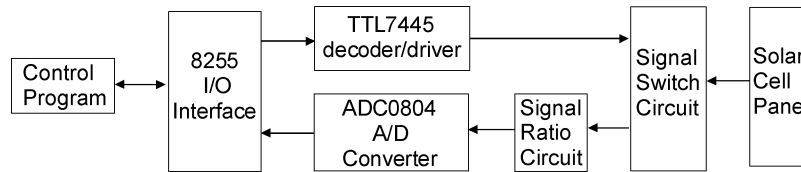


Fig. 7. Schematic diagram of the voltage measurement circuit.

order to ensure that the input signal of an analog-to-digital (A/D) converter is less than 5 V, the measured voltage amplitude needs to be reduced by a ratio of 0.25.

- 6) ADC0804 A/D converter: it is used to digitalize the voltage value measured from the solar-cell panel.

## V. SIMULATION RESULTS

The best solar-cell panel installation angle databases for different areas around Taiwan have been established by using GA and SA methods with the corresponding climatic data. The maximum number of photons that can be collected by the solar-cell panel is not only determined by the sunlight incident angle but also by the sunshine duration of each day. The information of sunshine duration for different days in a year is obtained from the surface data of the climatological data annual report published by the Central Weather Bureau of Taiwan. In order to simplify the problem, optical air mass changes and temperature effects of solar cells are neglected. Also, every solar-cell panel is oriented toward the south to normalize the climatological data. The number of the sunshine duration of each day is split into two segments and centered by the noon. For instance, if the sunshine duration number is 6 h, then, the time for collecting the photon on the surface of the solar-cell panel is from 9 in the morning to 3 in the afternoon. The photon density is accumulated every 5 min during the sunshine duration of the day with the corresponding sunlight incident angle.

Fig. 8 shows the optimal installation angle for each month from 1995 to 1998 at Chiayi, Taiwan, calculated by GA and SA techniques. For easy comparison, the specific values of both methods are listed in Table I. The solar-cell panel with a larger installation angle is more perpendicular to the horizon than the one with a smaller installation angle. For these months in January, February, November, and December, the solar elevation at noon and the sunshine duration are smaller than the ones in May, June, and July. Hence, larger installation angles are needed for the solar-cell panel to collect the maximum solar energy during the winter seasons.

The 1995 to 1998 annual solar energy in per-unit values for different solar-cell panel installation angles in the Chiayi areas is shown in Fig. 9. It can be found that the optimal installation angle for obtaining the maximum annual solar energy is about 20 degrees. The optimal installation angle of the fixed structure solar-cell panel for each year will vary slightly since the weather condition is different. The four-year average value will be the recommended answer for users to install the fixed structure solar-cell panels. With the same procedure, the solar-cell panel installation angle information database for different places in Taiwan can be established.

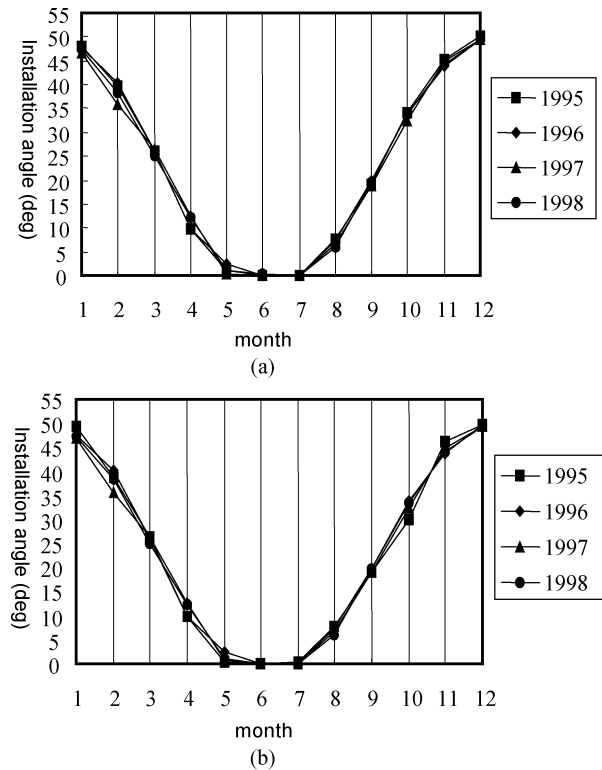


Fig. 8. Plots of the optimal installation angle for each month from 1995 to 1998 at Chiayi, Taiwan. (a) Angle obtained by the GA. (b) Angle obtained by the SA method.

TABLE I  
OPTIMAL INSTALLATION ANGLE FOR EACH MONTH FROM  
1995 TO 1998 AT CHIAYI, TAIWAN

month	1995		1996		1997		1998	
	GA	SA	GA	SA	GA	SA	GA	SA
Jan.	48.04	48.05	47.68	47.72	46.80	46.78	44.96	47.30
Feb.	39.85	39.88	40.21	40.24	35.72	35.67	38.18	38.15
Mar.	26.13	26.15	26.13	26.08	26.39	26.38	24.90	24.64
April	9.68	9.79	9.68	9.69	11.17	12.47	11.17	12.27
May	0.97	0.95	2.29	2.33	0.53	0.54	0.97	0.99
June	0.09	0.01	0.09	0.00	0.09	0.00	0.09	0.00
July	0.09	0.00	0.09	0.00	0.09	0.00	0.09	0.01
Aug.	7.83	7.92	7.48	7.48	6.69	6.71	5.81	5.86
Sept.	19.18	19.19	19.88	19.82	18.91	18.94	19.62	19.60
Oct.	34.05	34.05	33.70	33.69	32.46	32.61	33.70	33.66
Nov.	45.22	45.19	45.04	43.81	45.04	44.94	45.04	44.03
Dec.	50.06	50.07	49.53	49.53	49.35	49.39	49.35	49.36

In order to confirm the computer simulation results, hardware experiments are conducted for determining the optimal installation angle of the solar-cell panel. Five fixed structure solar-cell panels facing to the sharp south are installed with different installation angles, which are 10°, 20°, 30°, 40°, and 50°

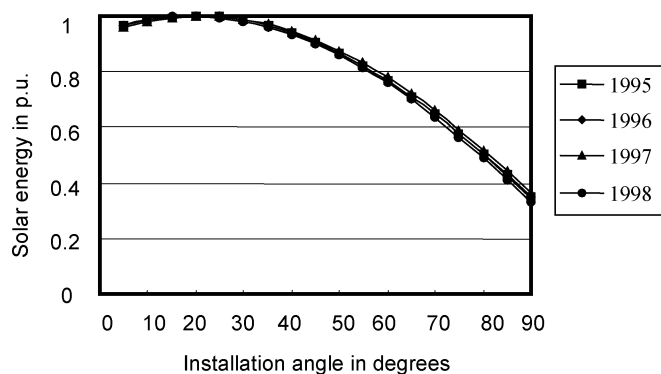


Fig. 9. The 1995 to 1998 annual solar energy in per-unit values for different solar-cell panel installation angles at Chiayi, Taiwan.

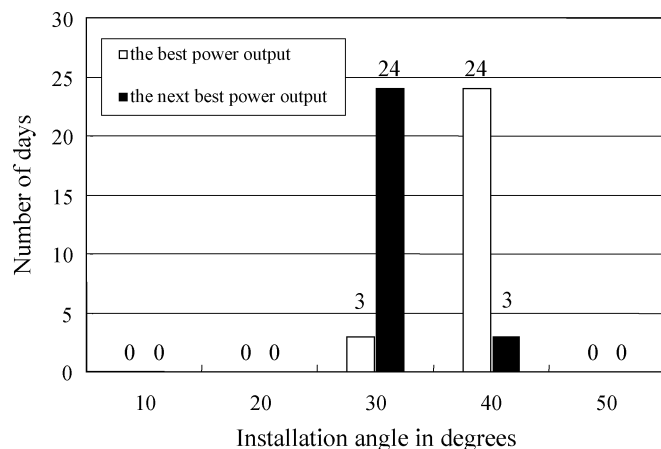


Fig. 10. Statistics histogram of the day number for five different installation angles with the best and the next best power output at Chiayi, Taiwan, in February 2001.

in winter seasons and  $0^\circ$ ,  $10^\circ$ ,  $20^\circ$ ,  $30^\circ$ , and  $40^\circ$  in summer seasons. A PC-based data-acquisition system was built to automatically record each solar cell panel's voltage, output current, and output power. Sequential data collections for each solar-cell panel are completed within 2 s. Since the weather condition does not change abruptly within 2 s, it is reasonable to assume that data for different solar-cell panels are collected simultaneously. The daily collected data of the solar-cell panel output power are screened by a software program to accumulate the valid ones and eliminate unvalued ones.

The objective of this experiment is to measure the actual output power from solar-cell panels with different installation angles and to make comparisons with the computer simulation results. Since the sunshine duration and the sunlight intensity for each day are different, the experimental results are represented in a relative manner (i.e., the optimal angle or the next optimal angle for the output power). It should be noted that data for rainy days are not included.

Fig. 10 shows the statistics histogram of the day number for five different installation angles with the best and the next best power output at Chiayi, Taiwan, in February 2001. The solar-cell panel with  $40^\circ$  has the highest day number for the best angle, and the one with  $30^\circ$  has the highest day number of the next best angle. In other words, Fig. 10 implies that the best angle is very close to  $40^\circ$ . The four-year-averaged computer

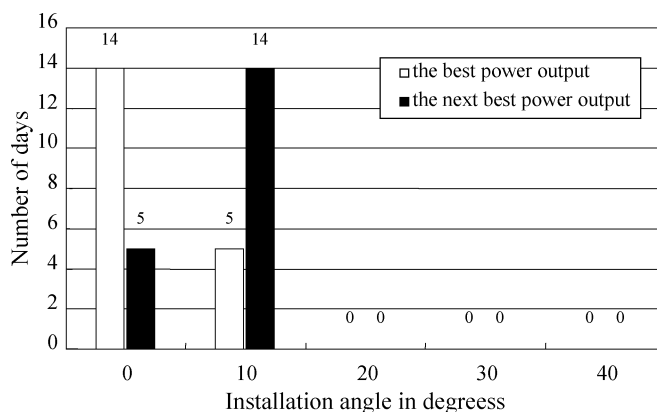


Fig. 11. Statistics histogram of the day number for five different installation angles with the best and the next best power output at Chiayi, Taiwan, in June 2001.

simulated best installation angle for February is about  $38.5^\circ$  for both GA and SA methods. It is very close to the hardware experimental results. It should be noted that differences of the accumulated power between the best power output and the next best power output for the same day are not significant. According to (5), the photon density ratio of two different surfaces is the cosine function of the included angle of the two surfaces. When the included angle is small, the ratio is close to 1. It means that these two photon density values are close to each other. However, the best and the next best output power are never shown on the other three installation angles.

Another similar statistics histogram for the month of June 2001 is shown in Fig. 11. Weeks of raining and cloudy days caused by two typhoons reduce the number of the valid data significantly. It was worse in July. However, the measured data still implied that the best installation angle for the solar-cell panel in this month is close to  $0^\circ$  because of its highest day number of the best power output. According to the computer simulation results, the best monthly installation angle at Chiayi, Taiwan, in June is  $0^\circ$  that agrees to the hardware experimental results.

## VI. CONCLUSION

This paper presents the calculation of the optimum installation angle of fixed solar-cell panels based on the GA and the SA method. In order to improve the efficiency of the fixed structure solar-cell panels, an optimum installation angle for solar-cell panels should be determined. The relationship between the sunlight incident angle and the sunlight radiation intensity on the solar-cell panel surface is presented in this paper. The best monthly and annual installation angles of fixed structure solar-cell panels for different areas in Taiwan are obtained by using GA and SA techniques with the corresponding weather data. Computer simulation results indicate that the optimal installation angles obtained by the GA or the SA method are very close to each other. It implies that both GA and SA are good techniques for determining the optimum installation angle for fixed solar-cell panels. Hardware experiments for solar-cell panels located at Chiayi, Taiwan, are conducted to confirm the computer simulation results. The experimental results indicate that the actual best monthly installation angles are very close to the computer simulation results.

## REFERENCES

- [1] B. Godfrey, *Renewable Energy: Power for a Sustainable Future*. Oxford, U.K.: Oxford Univ. Press, 1996.
- [2] M. Brower, *Cool Energy: Renewable Solutions to Environment Problems*. Cambridge, MA: MIT Press, 1993.
- [3] T. Markvart, *Solar Electricity*. New York: Wiley, 1994.
- [4] S. Roberts, *Solar Electricity: A Practical Guide to Designing and Installing Small Photovoltaic Systems*. Upper Saddle River, NJ: Prentice-Hall, 1991.
- [5] C. P. de Brichambaut, Cahiers A.F.E.D.E.S., supplement au no 1. Editions Europeennes Thermique et Industrie, Paris, France, 1975.
- [6] D. E. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning*. Reading, MA: Addison-Wesley, 1989.
- [7] Z. Michalewicz, *Genetic Algorithms + Data Structures = Evolution Programs*. New York: Springer-Verlag, 1992.
- [8] J. F. Frenzel, "Genetic algorithms—a new breed of optimization," *IEEE Potentials*, vol. 12, no. 3, pp. 21–24, Oct. 1993.
- [9] M. Mitchell, *An Introduction to Genetic Algorithms*. Cambridge, MA: MIT Press, 1996.
- [10] N. Metropolis, A. Rosenbluth, M. Rosenbluth, A. Teller, and E. Teller, "Equations of state calculations by fast computing machines," *J. Chem. Phys.*, vol. 21, pp. 1087–1092, 1953.
- [11] S. Kirkpatrick, C. D. Gelatt, Jr., and M. P. Vecchi, "Optimization by simulated annealing," *Sci.*, vol. 220, pp. 671–680, 1983.
- [12] G. E. Hinton and T. J. Sejnowski, "Learning and relearning in Boltzmann machines," in *Parallel Distributed Processing Volume 1*, D. E. Rumelhart and J. L. McClelland, Eds. Cambridge, MA: The MIT Press, 1986.
- [13] V. Cerny, "Thermodynamic approach to the traveling salesman problem: an efficient simulated algorithm," *J. Optim. Theory Appl.*, vol. 45, no. 1, 1985.



**Yaow-Ming Chen** (S'93–M'98) received the B.S. degree in electrical engineering from National Cheng Kung University, Tainan, Taiwan, R.O.C., in 1989 and the M.S. and Ph.D. degrees in electrical engineering from the University of Missouri, Columbia, in 1993 and 1997, respectively.

Currently, he is an Assistant Professor with the Department of Electrical Engineering, National Chung Cheng University, ChiaYi, Taiwan. He was an Assistant Professor from 1997 to 2000 with I-Shou University, Kaohsiung, Taiwan. His research interests include power-electronic converters, power system harmonics and compensation, and intelligent control.



**Chien-Hsing Lee** (S'93–M'98) was born in Pingtung, Taiwan, R.O.C., on June 13, 1967. He graduated from National Kaohsiung Institute of Technology, Taiwan, and received the B.S. degree in electrical engineering from Arizona State University, Tempe, in 1993. He received the M.S.E.E. and Ph.D. degrees from the Georgia Institute of Technology, Atlanta, in 1995 and 1998, respectively.

Currently, he is an Associate Professor with National Yunlin University of Science and Technology, Yunlin, Taiwan. His research interests are power system grounding analysis, power system transient modeling, power quality, and applications of wavelet theory in power systems.



**Hsu-Chin Wu** received the B.S. and M.S. degrees in electrical engineering from the I-Shou University, Kaohsiung, Taiwan, R.O.C., in 1999 and 2002, respectively.

Currently, he is with the Department of Electrical Engineering, Kangshan Vocational School, I-Shou University. His research interests include renewable energy, power application in industry, and artificial intelligence.